THE
[illegible]
LADIES and GENTLEMENS
DIARY,
OR,
ROYAL ALMANACK;
For the Year of our LORD, 1777:
Being the Firſt after Biſſextile, or Leap Year.

CONTAINING,
Beſides the CALENDAR, a great Variety of Ænigmas, Rebuſſes, Mathematical Solutions, &c. &c.

By REUBEN BURROW,
Late Aſſiſtant Aſtronomer at the Royal Obſervatory, and Teacher of the Mathematics.

LONDON:
Printed for T. CARNAN, in St. Paul's Church Yard, who diſpoſſeſſed the Stationers Company of the excluſive Privilege of Printing Almanacks, which they had monopolized 170 Years, to the diſcouragement of Genius and the great Prejudice of the Bookſellers throughout the kingdom, in Conſequence of a Patent obtained from King James I. which his moſt Sacred Majeſty had no Right to Grant.

ECLIPSES in 1777.

This year there will be Five Eclipſes, Three of the Sun, and Two of the Moon, which will happen in the following Order: The Firſt Eclipſe of the Sun will happen on the 9th of *January*, at Forty-Nine Minutes after Three in the Afternoon, only Part viſible.---The Second will be an Eclipſe of the Moon, beginning *January* 23d, at Forty-Seven Minutes after Two in the Afternoon, Middle Eleven Minutes after Four, ends Thirty-Six Minutes after Five, Digits eclipſed 7°. 6'. Moon riſes at Twenty-Five Minutes after Four, conſequently only Part viſible.---The Third Eclipſe will be of the Sun, *July* 4th, at Twenty-One Minutes paſt Midnight, inviſible.---The Fourth will be an Eclipſe of the Moon, *July* 20th, at Forty-Two Miuutes paſt Noon.---The Fifth is an Eclipſe of the Sun, which happens on the 29th of *December*, at Ten at Night, inviſible.

Common Notes, 1777.

Golden Number - -	10	Dominical Letter - -	E
Cycle of the Sun - -	22	Roman Indiction - -	19
Epact - - -	20	Number of Direction - -	9

The Four Quarters of the Year.

The Spring Quarter begins this Year the 20th of March, at 6 Hours 15 Minutes Morning, at which time the Sun enters *Equinoctial Sine Aries*, making equal Day and Night all the World over.

The Summer Quarter commences the 21ſt Day of June, at 4 Hours 33 Minutes, Morning, the Sun then entering into the *Sign Cancer*, making the longeſt Day to all the Northern, and the ſhorteſt to all the Southern Parts of the World.

The Autumnal Quarter begins the 22d Day of September, at 6 at Night, at which Time the Sun enters *Libra*, making again equal Day and Night to all Parts of the World.

The Winter Quarter begins the 21ſt of December, 10 Hours 20 Minutes, Morning, the Sun then entering into the *tropical Sign Capricorn*, making the ſhorteſt Day to the Northern, and longeſt to the Southern Inhabitants of the World.

WEIGHT and VALUE of the Gold and Silver Coins of England.

	Weight.		Value.		
GOLD.	dwt.	grs.	l.	s.	d.
A Guinea	5	9,438	1	1	0
Half Guinea	2	16,719	0	10	6
Quarter Guinea	1	8,359	0	5	3
SILVER.					
A Crown	19	8,519	0	5	0
Half Crown	9	16,259	0	2	6
Shilling	3	20,903	0	1	0
Sixpence	1	22,451	0	0	6

Current Gold Coin muſt weigh as follows:

	dwt.	grs.
Guineas	5	8
Half Guineas	2	16
Quarter Guineas	1	8

1777. January hath XXXI Days.

Laſt Quarter 1 day 9 h. 9 m. evening
New Moon 9 day 3 h. 39 m. afternoon
Firſt Quarter 16 day at noon
Full Moon 23 day 4 h. 19 m. afternoon
Laſt Quarter 31 day 6 h. 28 m. evening

Sun enters Aquarius
19d. 2h. 54m.
Apparent time.

1	W	Circumciſion	8	4	3	56	22	58	☽ riſes	23
2	TH	Mars riſes 11 36	8	4	3	56	22	52	0 m 11	24
3	F		8	3	3	57	22	46	1 14	25
4	S		8	2	3	58	22	40	2 19	26
5	E	2 S. aft. Chriſt. *O. Chriſt. d.*	8	1	3	59	22	33	3 24	27
6	M	Epiphany ***Twelfth Day***	8	0	4	0	22	26	4 35	28
7	TU		7	59	4	1	22	18	5 40	29
8	W	*Lucian*	7	58	4	2	22	10	6 45	30
9	TH	Sun eclipſed	7	57	4	3	22	1	☽ ſets	1
10	F		7	56	4	4	21	52	5 a 8	2
11	S		7	55	4	5	21	42	6 20	3
12	E	1 S. aft. Epiph. *O. N. Y. d.*	7	54	4	6	21	32	7 39	4
13	M	Camb. T. b. *Plow Mond.*	7	53	4	7	21	22	8 59	5
14	TU	Oxford Term begins	7	52	4	8	21	11	10 18	6
15	W		7	51	4	9	21	0	11 36	7
16	TH		7	50	4	10	20	49	Morn.	8
17	F	*Old Twelfth Day.*	7	49	4	11	20	37	0 56	9
18	S	Q. Ch. b. d. kept. Priſc.	7	48	4	12	20	24	2 14	10
19	E	2 S. aft. Epiph.	7	46	4	14	20	12	3 29	11
20	M	*Fabian* In 8 d. Hil. 1 Ret.	7	44	4	16	19	58	4 40	12
21	TU	*Agnes*	7	43	4	17	19	45	5 47	13
22	W	*Vincent*	7	42	4	18	19	31	6 43	14
23	TH	Hilary Term begins	7	41	4	19	19	17	☽ riſes	15
24	F		7	39	4	21	19	3	5 a 24	16
25	S	Converſion of St. Paul	7	37	4	23	18	48	6 29	17
26	E	Septuageſima Sunday	7	36	4	24	18	33	7 36	18
27	M	Pr. Aug. Fred. b. In 15	7	34	4	26	18	17	8 42	19
28	TU	[days of Hil. 2 Ret.	7	32	4	28	18	1	9 46	20
29	W		7	30	4	30	17	45	10 51	21
30	TH	King Charles beheaded	7	28	4	32	17	28	11 55	22
31	F		7	27	4	33	17	11	Morn.	23

Days	Leng. of Days.	Days increaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	7 52	0 8	5 59	4 41	6 1	4 20	8 A 43
6	8 0	0 16	5 56	4 43	6 4	6 36	8 20
11	8 10	0 26	5 53	4 47	6 8	8 41	7 58
16	8 20	0 37	5 48	4 50	6 12	10 29	7 37
21	8 34	0 50	5 43	4 55	6 17	11 59	7 16
26	8 49	1 5	5 37	4 58	6 23	13 10	6 55

February hath XXVIII Days. 1777

New Moon	8 day	4 h.	32 m.	morning	Sun enters Piſces.
Firſt Quarter	14 day	8 h.	18 m.	night	17d. 17h. 46m.
Full Moon	22 day	9 h.	19 m.	morning	Apparent time.

D M	D W	Sundays, Holidays, &c.	☉ riſes.		☉ ſets.		☉'s declin.		☽ riſes & ſets.		☽'s age.
1	S		7	26	4	34	16	54S	1 m	0	24
2	E	Candlemas d. Sexag. S.	7	24	4	36	16	37	2	7	25
3	M	*Blaſe* Mor. Purif. 3 Ret.	7	22	4	38	16	19	3	13	26
4	Tu	Mars riſes 10 11	7	20	4	40	16	1	4	19	27
5	W	*Agatha*	7	18	4	42	15	43	5	19	28
6	Th		7	16	4	44	15	24	6	15	29
7	F	Venus ſets 8 43	7	14	4	46	15	5	☽ ſets		1
8	S		7	12	4	48	14	46	5 a	9	2
9	E	Shrove Sunday	7	11	4	49	14	27	6	32	3
10	M	In 8 days of Pur. 4 Ret.	7	10	4	50	14	7	7	54	4
11	Tu	Shrove Tueſday	7	8	4	52	13	48	9	19	5
12	W	Aſh Wedn. Hil. T. ends	7	6	4	54	13	28	10	41	6
13	Th	Old Candlemas day	7	5	4	55	13	7	11	59	7
14	F	*Valentine*	7	3	4	57	12	47	morn.		8
15	S	Camb. Term divides	7	1	4	59	12	26	1	18	9
16	E	1 Sunday in Lent	6	59	5	1	12	6	2	29	10
17	M		6	57	5	3	11	44	3	37	11
18	Tu		6	55	5	5	11	23	4	36	12
19	W	Ember Week	6	53	5	7	11	2	5	26	13
20	Th		6	51	5	9	10	40	6	7	14
21	F		6	49	5	11	10	19	☽ riſes		15
22	S	Procyon S. 9 1	6	47	5	13	9	57	5 a	23	16
23	E	2 Sunday in Lent	6	45	5	15	9	35	6	29	17
24	M	St. Matthias, Pr. Adol.	6	43	5	17	9	12	7	35	18
25	Tu	[Fr. born	6	41	5	19	8	50	8	39	19
26	W		6	39	5	21	8	28	9	44	20
27	Th		6	38	5	22	8	5	10	49	21
28	F		6	36	5	24	7	42	11	54	22

Days.	Leng. of Days.		Days increaſe.		Day breaks.		Sun Eaſt.		Twilight ends.		Clock before Sun		Seven Stars South.	
1	9	9	1	25	5	28	5	5	6	32	14	8	6 A	31
6	9	27	1	43	5	21	5	10	6	39	14	34	6	11
11	9	45	2	1	5	12	5	16	6	48	14	41	5	51
16	10	3	2	19	5	4	5	21	6	56	14	27	5	31
21	10	23	2	39	4	55	5	27	7	5	13	56	5	12
26	10	43	2	59	4	46	5	33	7	14	13	9	4	54

1777. March hath XXXI Days.

Laſt Quarter 2 day 1 h. 42 m. afternoon.
New Moon 9 day 3 h. 20 m. afternoon.
Firſt Quarter 16 day 6 h. 11 m. morning.
Full Moon 24 day 2 h. 54 m. morning.

S. enters Aries.
19d. 18h. 15m.
Apparent time.

1	S	*David*,	6 34	5 26	7 20S	Morn.	23
2	E	**3 Sunday in Lent.** Chad.	6 32	5 28	6 57	1 2	24
3	M		6 30	5 30	6 34	2 6	25
4	Tu	Jupiter South 8 3	6 28	5 32	6 11	3 8	26
5	W		6 26	5 34	5 48	4 4	27
6	Th		6 24	5 36	5 24	4 53	28
7	F	*Perpetua*	6 22	5 38	5 1	5 36	29
8	S		6 20	5 40	4 38	6 10	30
9	E	Midlent Sunday	6 18	5 42	4 14	☽ ſets	1
10	M		6 16	5 44	3 51	6 a 56	2
11	Tu		6 14	5 46	3 27	8 22	3
12	W	*Gregory* M.	6 12	5 48	3 3	9 46	4
13	Th	Regulus South 10 20	6 10	5 50	2 40	11 7	5
14	F		6 8	5 52	2 16	Morn.	6
15	S		6 6	5 54	1 52	0 27	7
16	E	5 Sunday in Lent	6 4	5 56	1 29	1 38	8
17	M	*St. Patrick*	6 2	5 58	1 5	2 40	9
18	Tu	*Edward K. W. S.*	6 0	6 0	0 41	3 32	10
19	W	Mars riſes 7 7	5 58	6 2	0 18	4 15	11
20	Th	Equal Day and Night	5 56	6 4	0 5N	4 48	12
21	F	Benedict. Camb. T. ends	5 54	6 6	0 29	5 19	13
22	S	Oxford Term ends	5 52	6 8	0 53	5 42	14
23	E	6 Sun. in Lent. Palm S.	5 50	6 10	1 16	☽ riſes	15
24	M		5 48	6 12	1 40	6 a 36	16
25	Tu	Lady-day	5 46	6 14	2 3	7 41	17
26	W		5 44	6 16	2 27	8 47	18
27	Th	*Maund. Thurſday*	5 43	6 17	2 50	9 52	19
28	F	**Good Friday**	5 41	6 19	3 14	10 59	20
29	S		5 39	6 21	3 37	Morn.	21
30		Eaſter Sunday	5 37	6 23	4 0	0 5	22
31	M	**Eaſter Monday**	5 35	6 25	4 23	1 7	23

Days	Leng. of Days	Days increaſe	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	10 53	3 9	4 41	5 37	7 19	12 34	4 A42
6	11 13	3 29	4 31	5 43	7 29	11 26	4 24
11	11 33	3 49	4 20	5 50	7 40	10 9	4 5
16	11 53	4 9	4 9	5 56	7 51	8 43	3 47
21	12 13	4 29	3 58	6 1	8 2	7 12	3 29
[illegible]	[illegible]	[illegible]	[illegible]	6 7	8 14	5 20	3 11

April hath XXX Days. 1777.

Laſt Quarter	1 day	5 h. 31 m.	morning.	
New Moon	7 day	0 h. 18 m.	midnight.	Sun enters Taurus.
Firſt Quarter	14 day	6 h. 1 m.	afternoon.	19d. 7h. 7m.
Full Moon	22 day	7 h. 52 m.	evening.	Apparent time.
Laſt Quarter	30 day	5 h. 18 m.	afternoon.	

1	Tu	**Eaſter Tueſday**	5 33	6 27	4 47N	2 m 4	24
2	W		5 31	6 29	5 10	2 54	25
3	Th	*R. Biſhop Chicheſter*	5 29	6 31	5 33	3 37	26
4	F	*St. Ambroſe*	5 27	6 33	5 55	4 15	27
5	S	*Old Lady-day*	5 25	6 35	6 18	4 46	28
6	E	**1 S. after Eaſt. Low S.**	5 23	6 37	6 41	5 12	29
7	M	Saturn riſes 7 37	5 21	6 39	7 3	☽ ſets	1
8	Tu		5 19	6 41	7 26	7 a 21	2
9	W	Oxf. and Camb. T. beg.	5 17	6 43	7 48	8 47	3
10	T		5 15	6 45	8 10	10 13	4
11	F		5 13	6 47	8 32	11 30	5
12	S		5 11	6 49	8 54	Morn.	6
13	E	**2 Sunday after Eaſter**	5 10	6 50	9 16	0 40	7
14	M	From Eaſt. in 2 weeks 1	5 8	6 52	9 37	1 37	8
15	Tu	[Ret.	5 6	6 54	9 59	2 24	9
16	W	**Eaſter Term begins**	5 4	6 56	10 20	3 1	10
17	Th		5 2	6 58	10 41	3 32	11
18	F		5 0	7 0	11 2	3 55	12
19	S	*Alphege*	4 58	7 2	11 23	4 19	13
20	E	**3 Sunday after Eaſter**	4 56	7 4	11 43	4 35	14
21	M	From Eaſt. in 3 weeks 2	4 54	7 6	12 4	4 54	15
22	T	[Ret.	4 52	7 8	12 24	5 12	16
23	W	**St. George**	4 50	7 10	12 44	☽ riſes	17
24	T		4 49	7 11	13 3	8 a 54	18
25	F	**St. Mark. Prs. M. born**	4 47	7 13	13 23	10 5	19
26	S		4 45	7 15	13 42	11 8	20
27	E	**4 Sunday after Eaſter**	4 43	7 17	14 1	Morn.	21
28	M	From Eaſt. in 4 weeks	4 41	7 19	14 20	0 8	22
29	Tu	[3 Ret.	4 40	7 20	14 39	1 2	23
30	W		4 38	7 22	14 57	1 43	24

Days	Leng. of days.	Days increaſe.	Day breaks.	Sun Faſt	Twilight ends.	Clock before Sun.	Seven Stars South.
1	12 55	5 11	3 31	6 15	8 29	3 47	2 A 50
6	13 15	5 31	3 18	6 21	8 42	2 18	2 32
11	13 35	5 51	3 5	6 27	8 55	0 54	2 13
16	13 53	6 9	2 51	6 33	9 9	0 A 23	1 55
21	14 13	6 29	2 36	6 39	9 24	1 31	1 37
26	14 31	6 47	2 20	6 44	9 40	2 28	1 18

May hath XXXI Days.

New Moon 7 day 8 h. 8 m, morning.	
Firſt Quarter 14 day 7 h. 46 m. morning.	Sun enters Gemini.
Full Moon 22 day 11 h. 24 m. morning.	20d. 7h. 47m.
Laſt Quarter 30 day 1 h. 18 m. morning.	Apparent time.

1	Th	**St. Phil. and Ja.**	4	36	7	24	15	15N	2 m 22	25
2	F		4	34	7	26	15	33	2 53	26
3	S	*Inv. of the Croſs*	4	33	7	27	15	51	3 20	27
4	**E**	Rogation Sunday	4	31	7	29	16	8	3 45	28
5	M	From Eaſt. in 5 weeks 4	4	30	7	30	16	25	4 8	29
6	Tu	*John à P. Lat.* [Ret.	4	28	7	32	16	42	☽ ſets	1
7	W		4	26	7	34	16	58	7 a 47	2
8	Th	**Aſcenſ. day. Holy Thurſ.**	4	25	7	35	17	15	9 9	3
9	F	On mor. of Aſcenſ. 5 Ret.	4	23	7	37	17	31	10 24	4
10	S		4	22	7	38	17	46	11 32	5
11	**E**	S. after Aſcenſion day	4	20	7	40	18	2	Morn.	6
12	M	**Term ends.** *Old May-day*	4	18	7	43	18	17	0 24	7
13	Tu		4	17	7	42	18	32	1 5	8
14	M		4	16	7	44	18	46	1 37	9
15	Th	Oxford Term ends	4	14	7	46	19	0	2 5	10
16	F		4	12	7	48	19	14	2 27	11
17	S	Arcturus South 10 25	4	11	7	49	19	27	2 46	12
18	**E**	**Whit-Sunday**	4	10	7	50	19	41	3 4	13
19	M	**Q. Cha. b. 1744.** *Dunſt.*	4	8	7	52	19	54	3 22	14
20	Tu	**Whit-Tueſday**	4	6	7	54	20	6	3 40	15
21	W	**Ember Week**	4	5	7	55	20	18	4 0	16
22	Th	**Prs. Eliz. born**	4	3	7	57	20	30	☽ riſes	17
23	F		4	2	7	58	20	42	9 a 1	18
24	S	*No N. but twil. till July* 21.	4	1	7	59	20	53	10 3	19
25	**E**	**Trinity Sunday**	4	0	8	0	21	3	10 59	20
26	M	*Auguſtin.* Mor. of Tr.	3	59	8	1	21	14	11 47	21
27	Tu	*Ven. Bede* [1 Ret.	3	58	8	2	21	24	Morn.	22
28	W	Oxford Term begins	3	57	8	3	21	34	0 24	23
29	Th	**K. Ch. II. Reſt.** *C. Chriſti*	3	56	8	4	21	43	0 57	24
30	F	**Trinity Term begins**	3	55	8	5	21	52	1 24	25
31	S		3	54	8	6	22	0	1 47	26

Days	Leng. o. Days.	Days in creaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock af ter Sun.	Seven Stars South.
1	14 49	7 5	2 3	6 50	9 57	3 12	0 A 59
6	15 5	7 21	1 46	6 55	10 14	3 43	0 40
11	15 21	7 37	1 24	7 0	10 36	3 58	0 20
16	15 36	7 52	1 0	7 4	11 0	4 0	0 0
21	15 50	8 6	0 18	7 8	11 42	3 48	11 M 40
26	16 2	8 18	No Night	7 12	No Night	3 23	11 20

June hath XXX Days.

New Moon 5 day 3 h. 48 m. afternoon.
Firſt Quarter 12 day 11 h. 2 m. night.
Full Moon 21 day 1 h. 5 m. morning.
Laſt Quarter 28 day 6 h. 38 m. morning.

S. enters Gemini.
20d. 16h. 33m.
Apparent time.

1	E	1 S. aft. Trin. Nicom.	3 53	8 7	22 8N	2 m12	27
2	M	In 1 week aft. Tr. 2 Ret.	3 52	8 8	22 16	2 33	28
3	Tu		3 51	8 9	22 24	2 55	29
4	W	K. George III. born	3 51	8 10	22 31	3 23	30
5	Th	Pr. Er. Aug. b. *Boniface*	3 50	8 11	22 37	☽ ſets	1
6	F	Lyra South 1 32	3 49	8 12	22 43	9 a 10	2
7	S		3 48	8 12	22 49	10 11	3
8	E	2 Sunday after Trinity	3 48	8 13	22 55	10 59	4
9	M	In 2 weeks aft. Tr. 3 Ret.	3 47	8 13	23 0	11 35	5
10	Tu	Princeſs Amelia born	3 47	8 14	23 4	Morn.	6
11	W	St. Barnabas	3 46	8 15	23 8	0 3	7
12	Th		3 45	8 15	23 12	0 28	8
13	F		3 45	8 16	23 16	0 49	9
14	S	Clock with the ſun	3 44	8 16	23 18	1 8	10
15	E	3 Sunday after Trinity	3 44	8 16	23 21	1 24	11
16	M	In 3 weeks aft. Tr. 4 Ret.	3 44	8 17	23 23	1 43	12
17	Tu	*St. Alban*	3 43	8 17	23 25	2 1	13
18	W	Trinity Term ends	3 43	8 17	23 26	2 23	14
19	Th		3 43	8 17	23 27	2 49	15
20	F	*T. Edw. K. W. S.*	3 43	8 17	23 28	3 22	16
21	S	Long. day 16 34	3 43	8 17	23 28	☽ riſes	17
22	E	4 Sunday after Trinity	3 43	8 17	23 27	9 a 41	18
23	M		3 43	8 17	23 26	10 23	19
24	Tu	St. John Bapt.	3 43	8 17	23 25	10 57	20
25	W		3 44	8 16	23 24	11 24	21
26	Th		3 44	8 16	23 22	11 48	22
27	F		3 44	8 16	23 20	Morn.	23
28	S		3 44	8 16	23 17	0 10	24
29	E	5 S. aft. Trin. St. Peter	3 45	8 15	23 13	0 33	25
30	M		3 45	8 15	23 10	0 54	26

Days	Leng. of Days.	Days in-creaſe.	Day breaks.	Sun Eaſt	Twiligh ends.	Clock af-ter Sun.	SevenStars South.
1	16 14	8 32	No real night, bu constant twilight	7 16	No real night, bu constant twilight	2 37	10 M 56
6	16 22	8 38		7 18		1 47	10 35
11	16 28	8 44		7 19		0 49	10 15
16	16 32	8 48		7 20		bef. 13	9 54
21	16 34	8 50		7 20		1 18	9 33
26	16 32	dec. 2					

1777. July hath XXXI Days.

New Noon 4 day 0 h. 21 m. midnight.
Firſt Quarter 12 day 3 h. 34 m. afternoon.
Full Moon 20 day 0 h. 52 m. afternoon.
Laſt Quarter 27 day 10 h. 55 m. morning.

Sun enters Leo.
22d. 3h. 23m.
Apparent time.

1	Tu	Camb. Comm.	3 46	8 14	23 6N	1 m 18	27
2	W	*Viſit. B. V. M.*	3 46	8 14	23 1	1 50	28
3	Th		3 47	8 13	22 57	2 27	29
4	F	Camb. T. ends. *Tran. St*	3 48	8 12	22 51	☽ ſets	1
5	S	*Old Midſ. day* [*Mar.*	3 48	1 12	22 46	8 a 42	2
6	E	6 Sunday after Trinity	3 49	8 11	22 40	9 30	3
7	M	Oxford Act	3 50	8 10	22 33	10 0	4
8	Tu		3 50	8 10	22 26	10 25	5
9	W		3 51	8 9	22 19	10 48	6
10	Th		3 52	8 8	22 12	11 7	7
11	F		3 53	8 7	22 4	11 25	8
12	S		3 54	8 6	21 55	11 43	9
13	E	7 Sunday after Trinity	3 55	8 5	21 47	Midn.	10
14	M		3 56	8 4	21 38	Morn.	11
15	Tu	*Swithin*	3 57	8 3	21 28	0 21	12
16	W		3 58	8 2	21 18	0 44	13
17	Th		3 59	8 1	21 8	1 14	14
18	F		4 0	8 0	20 58	1 52	15
19	S	Oxford Term ends	4 1	7 59	20 47	2 38	16
20	E	8 S. aft. Trin.	4 3	7 57	20 35	☽ riſes	17
21	M	Dog-days begin	4 4	7 56	20 24	8 a 53	18
22	Tu	*St. Mary Magdalen*	4 5	7 55	20 12	9 24	19
23	W		4 6	7 54	20 0	9 50	20
24	Th		4 8	7 52	19 47	10 13	21
25	F	St. James	4 10	7 50	19 34	10 36	22
26	S	*St. Anne, Mother of V. M.*	4 11	7 49	19 21	10 57	23
27	E	9 Sunday after Trinity	4 13	7 47	19 7	11 23	24
28	M		4 14	7 46	18 53	11 47	25
29	Tu		4 15	7 45	18 39	Morn.	26
30	W		4 17	7 43	18 24	0 21	27
31	Th		4 18	7 42	18 10	1 4	28

Days	Leng. of days.	Days increaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	Seven Stars Sonth.
1	16 28	0 6		7 19		3 21	8 M 52
6	16 22	0 12	No real	7 17	No real	4 16	8 31
11	16 14	0 20	Night.	7 15	Night.	5 2	8 11
16	16 4	0 30		7 12		5 35	7 50
21	15 52	0 42	0 24	7 8	11 36	5 55	7 30

Auguſt hath XXXI Days. 1777.

New Moon 3 day 10 h. 45 m. morning. First Quarter 11 day 9 h. 0 m. morning. **Full Moon** 18 day 11 h. 8 m. night. Laſt Quarter 25 day 3 h. 54 m. afternoon.	Sun enters Virgo. 22d. 9h. 39m.

1	F	**Lammas-day**	4 19	7 41	17 54N	1m 54	29
2	S		4 21	7 39	17 39	☽ ſets	1
3	E	**10 Sunday after Trinity**	4 23	7 37	17 23	7 a 57	2
4	M		4 24	7 36	17 7	8 27	3
5	Tu		4 25	7 35	16 51	8 51	4
6	W	*Transfiguration*	4 27	7 33	16 35	9 10	5
7	Th	*Name of Jeſus*	4 28	7 32	16 18	9 30	6
8	F		4 30	7 30	16 1	9 47	7
9	S		4 31	7 29	15 43	10 6	8
10	E	**11 S. aft. Trin.** St. Law-	4 33	7 27	15 26	10 25	9
11	M	**Prs. Brunſw. b.** [rence	4 35	7 25	15 8	10 45	10
12	Tu	**Pr. Wales b.** Old Lam. d.	4 36	7 24	14 50	11 14	11
13	W		4 38	7 22	14 32	11 48	12
14	Th		4 40	7 20	14 13	Morn.	13
15	F	*Aſſumption Virgin Mary*	4 42	7 18	13 54	0 29	14
16	S	**Prince Fred. born, 1763**	4 44	7 16	13 35	1 22	15
17	E	**12 Sunday after Trinity**	4 45	7 15	13 16	2 26	16
18	M		4 47	7 13	12 57	3 37	17
19	Tu		4 49	7 11	12 37	☽ riſes	18
20	W		4 51	7 9	12 17	8 19	19
21	Th	Pr. Wm. Henry b. 1765	4 53	7 7	11 57	8 43	20
22	F		4 54	7 6	11 37	9 6	21
23	S		4 56	7 4	11 17	9 28	22
24	E	**13 S. aft. Trin.** St. Bar-	4 58	7 2	10 56	9 56	23
25	M	[tholomew	5 0	7 0	10 35	10 27	24
26	Tu		5 2	6 58	10 14	11 6	25
27	W		5 4	6 56	9 53	11 52	26
28	Th	*St. Auſtin*	5 6	6 54	9 32	Morn.	27
29	F	*Dec. St. John Baptiſt*	5 8	6 52	9 11	0 50	28
30	S		5 10	6 50	8 49	1 56	29
31	E	**14 Sunday after Trinity**	5 12	6 48	8 27	3 4	30

Days	Leng. of days.	Days decreaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock before Sun.	SevenStars South.
1	15 20	1 14	1 29	6 59	10 31	5 50	6M47
6	15 5	1 29	1 49	6 54	10 11	5 25	6 28
11	14 49	1 45	2 7	6 49	9 53	4 45	6 9
16	14 31	2 3	2 25	6 44	9 35	3 50	5 50
21	14 13	2 21	2 39	6 38	9 21	2 42	5 32
26	13 55	2 39	2 52	6 33	9 8	1 23	5 13

1777. September hath XXX Days.

New Moon 1 day 11 h. 32 m. night.
Firſt Quarter 10 day 2 h. 41 m. morning.
Full Moon 17 day 8 h. 24 m. morning.
Laſt Quarter 23 day 11 h. 3 m. night.

Sun enters Libra.
22d. 6h. om.
Apparent time.

1	M	*ries* Dog-days end	5 14	6 46	8 6	☽ ſets	1
2	Tu	London burnt, 1666	5 16	6 44	7 44	7 a 21	2
3	W		5 17	6 43	7 22	7 40	3
4	Th		5 19	6 41	6 59	7 58	4
5	F		5 21	6 39	6 37	8 17	5
6	S	Fomalhaut South 11 40	5 23	6 37	6 15	8 36	6
7	**E**	15 S. aft. Trin. *Erurchus*	5 25	6 35	5 52	8 56	7
8	M	*Nativity V. Mary*	5 27	6 33	5 29	9 22	8
9	Tu		5 29	6 31	5 7	9 52	9
10	W		5 31	6 29	4 44	10 30	10
11	Th		5 33	6 27	4 21	11 16	11
12	F		5 35	6 25	3 58	Morn.	12
13	S	Venus riſes 1 35	5 37	6 23	3 35	0 12	13
14	**E**	16 S. aft. Trin. *Holy-*	5 39	6 21	3 12	1 22	14
15	M	[*croſs-day*	5 41	6 19	2 49	2 36	15
16	Tu		5 43	6 17	2 26	3 58	16
17	W	Ember Week. *Lamb.*	5 45	6 15	2 2	5 21	17
18	Th		5 47	6 13	1 39	☽ riſes	18
19	F		5 49	6 11	1 16	7 a 39	19
20	S		5 51	6 9	0 52	8 6	20
21	**E**	17S. aft. Tr. St. Matthew	5 53	6 7	0 29	8 36	21
22	M	K. George crowned 1761	5 55	6 5	0 6	9 13	22
23	Tu		5 57	6 3	0 17S	9 58	23
24	M		5 59	6 1	0 41	10 53	24
25	Th		6 1	5 59	1 4	11 55	25
26	F	*St. Cyprian*	6 3	5 57	1 28	Morn.	26
27	S		6 5	5 55	1 51	1 4	27
28	**E**	18 S. after Trinity	6 7	5 53	2 14	2 14	28
29	M	St. Mich. Prs. Ch. Aug.	6 9	5 51	2 38	3 23	29
30	Tu	*St. Jerome* [bo. 1766	6 11	5 49	3 1	4 42	30

Days	Leng. of days.	Days decreaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock after Sun.	Seven Stars South.
1	13 31	3 3	3 10	6 25	8 50	0 23	4 M 52
6	13 13	3 21	3 23	6 19	8 37	1 59	4 34
11	12 53	3 41	3 38	6 13	8 22	3 41	4 16
16	12 33	4 1	3 47	6 7	8 13	5 26	3 58
21	12 13	4 21	3 59	6 1	8 1	7 11	3 40
26	11 53	4 41	4 10	5 55	7 50	8 53	3 22

October hath XXXI Days.

New Moon 1 day 2 h. 57 m. afternoon. Firſt Quarter 9 day 7 h. 34 m. evening. Full Moon 16 day 5 h. 27 m. evening. Laſt Quarter 23 day 9 h. 28 m. morning. New Moon 31 day 8 h. 33 m. morning.	Sun enters Scorpio. 22d. 13h. 51m. Apparent time.

1	W	*Remigius*	6 12	5 48	3 25S	☽ ſets	1
2	Th		6 14	5 46	3 48	6 a 31	2
3	F	Fomalhaut South 10 3	6 16	5 44	4 11	6 49	3
4	S		6 18	5 42	4 34	7 8	4
5	**E**	**19 S. after Trin.**	6 20	5 40	4 58	7 32	5
6	M	*Faith*	6 22	5 38	5 21	8 0	6
7	Tu	Jupiter riſes 1 18	6 24	5 36	5 44	8 34	7
8	W		6 26	5 34	6 7	9 17	8
9	Th	*St. Denys*	6 28	5 32	6 30	10 7	9
10	F	**Oxf. & Cam. Term. beg.**	6 30	5 30	6 52	11 8	10
11	S	[*Old Mich. day*	6 32	5 28	7 15	Morn.	11
12	**E**	**20 Sunday after Trin.**	6 34	5 26	7 38	0 20	12
13	M	*Tr. K. Ed. Confeſſor*	6 36	5 24	8 0	1 36	13
14	Tu		6 38	5 22	8 22	2 58	14
15	W		6 40	5 20	8 45	4 23	15
16	Th		6 42	5 18	9 7	☽ riſes	16
17	F	*Ethelred*	6 44	5 16	9 29	6 11	17
18	S	St. Luke	6 46	5 14	9 51	6 41	18
19	**E**	**21 Sund. aft. Trin**	6 47	5 13	10 12	7 15	19
20	M		6 49	5 11	10 34	7 57	20
21	Tu		6 51	5 9	10 55	8 52	21
22	W		6 53	5 7	11 17	9 50	22
23	Th		6 55	5 5	11 38	10 59	23
24	F		6 57	5 3	11 59	Morn.	24
25	S	**K. Geo. III. Ac.** *Criſpin*	6 59	5 1	12 19	0 11	25
26	**E**	**22 S. aft. Tr. King Geo**	7 1	4 59	12 40	1 20	26
27	M	[Proc	7 3	4 57	13 0	2 28	27
28	Tu	St. Simon and Jude	7 5	4 55	13 20	3 37	28
29	W	Fomalhaut South 8 26	7 7	4 53	13 40	4 42	29
30	Th		7 9	4 51	14 0	☽ ſets	1
31	F		7 11	4 49	14 20	5 a 10	2

Day	Leng. of days.	Days de-creaſe.	Day breaks.	Sun Eaſt	Twilight ends.	Clock after Sun.	Seven Star South.
1	11 35	4 59	4 20	5 49	7 40	10 30	3 M 4
6	11 15	5 19	4 31	5 43	7 29	11 59	2 46
11	10 55	5 39	4 40	5 36	7 20	13 19	2 28
16	10 35	5 59	4 50	5 30	7 10	14 26	2 10
21	10 17	6 17	4 59	5 24	7 1	15 19	1 51

November hath XXX. Days.

Firſt Quarter 8 day 10 h. 31 m. morning
Full Moon 15 day 3 h. 3 m. morning
Laſt Quarter 21 day 11 h. 32 m. night
New Moon 30 day 3 h. 24 m. morning

S. ent. Sagittarius
21d. 9h. 59m.
Apparent time.

1	S	All Saints	7	12	4	48	14	39S	5 a	39	3
2	E	23 S. aft. Tr. Pr. Ed. bo	al!So		4	46	14	58	6	6	4
3	M	Mor. of all Souls 1 Ret.	7	16	4	44	15	17	6	36	5
4	Tu		7	18	4	42	15	35	7	15	6
5	W	Powder Plot, 1605	7	20	4	40	15	53	8	1	7
6	Th	*Leonard.*	7	21	4	39	16	11	8	58	8
7	F	D. of Cum. b. 1745	7	23	4	37	16	29	10	4	9
8	S	Prs. Auguſ. So. bo. 1768	7	24	4	36	16	46	11	15	10
9	E	24 S. aft. Trin. Ld. May.	7	26	4	34	17	4	morn.		11
10	M	[day	7	28	4	32	17	20	0	33	12
11	Tu	*St. Martin*	7	30	4	30	17	37	1	54	13
12	W	Mor. of St. Mart. 2 Ret.	7	31	4	29	17	53	3	18	14
13	Th	*Britius*	7	32	4	28	18	9	4	41	15
14	F		7	34	4	26	18	25	☽ riſes		16
15	S	*Machutus*	7	36	4	24	18	40	5 a	6	17
16	E	25 S. after Trinity	7	37	4	23	18	55	5	46	18
17	M	*Hugh B. Lincoln*	7	38	4	22	19	10	6	35	19
18	Tu	In 8 days of St. Mar. 3 Ret.	7	40	4	20	19	24	7	34	20
19	W		7	42	4	18	19	38	8	40	21
20	Th	*Edw. K. and Mart.*	7	44	4	16	19	52	9	53	22
21	F		7	45	4	15	20	5	11	4	23
22	S	*Old Mart. Day. Cecilia*	7	46	4	14	20	18	morn.		24
23		26 S. after Trin. *Clement*	7	48	4	12	30	30	0	15	25
24	M		7	49	4	11	20	42	1	25	26
25	Tu	D. of Glouceſt. b. 4 Ret.	7	50	4	10	20	54	2	30	27
26	W		7	51	4	9	21	5	3	36	28
27	Th		7	52	4	8	21	16	4	41	29
28	F	Michaelm. Term ends	7	53	4	7	21	27	5	48	30
29	S		7	54	4	6	21	37	☽ ſets		1
30	E	Advent S. St. Andrew	7	55	4	5	21	46	4 a	33	2

Days.	Leng. of Days.		Day decreaſe.		Day breaks.		un Eaſt.		Twilight ends.		Clock after Sun.		Seven Stars ſouth.	
1	9	35	6	59	5	17	5	17	6	43	16	13	1 M	8
6	9	17	7	17	5	26	5	6	6	34	16	[illegible]	0	48
11	9	0	7	34	5	32	5	1	6	28	15	38	0	28
16	8	45	7	49	5	38	4	56	6	22	14	50	0	8
21	8	30	8	4	5	44	4	52	6	16	13	40	11 A	43
26	8	18	8	16	5	49	4	48	6	11	12	11	11	20

December hath XXXI. Days. 1777.

Firſt Quarter	7 day	10 h. 45 m. night	
Full Moon	14 day	1 h. 39 m. aftern.	S. ent. Capricorn
Laſt Quarter	21 day	5 h. 5 m. aftern.	20d. 22h. 20m.
New Moon	29 day	10 h. 0 m. night	Apparent time.

1	M		7	57	4	3	21	56	5	a 7	3
2	Tu		7	58	4	2	22	5	5	51	4
3	W		7	59	4	1	22	13	6	44	5
4	Th	Aldebaran South 11 35	8	0	4	0	22	21	7	47	6
5	F		8	0	4	0	22	29	8	55	7
6	S	*Nicholas*	8	1	3	59	22	36	10	10	8
7	E	**2 Sunday in Advent**	8	2	3	58	22	42	11	26	5
8	M	*Conception Virgin Mary*	8	3	3	57	22	49	morn.		10
9	Tu		8	4	3	56	22	54	0	43	11
10	W		8	5	3	55	23	0	2	3	12
11	Th		8	5	3	55	23	5	3	25	13
12	F		8	6	3	54	23	9	4	52	14
13	S	*Lucy*	8	6	3	54	23	13	6	17	15
14	E	**3 Sunday in Advent**	8	6	3	54	23	16	☽ riſes		16
15	M		8	7	3	53	23	19	5	a 2	17
16	Tu	O Sap. Camb Term ends	8	7	3	53	23	22	6	8	18
17	W	Emb. Week. Oxf. Term	8	7	3	53	23	24	7	20	19
18	Th	[ends	8	8	3	52	23	26	8	35	20
19	F		8	8	3	52	23	27	9	45	21
20	S	Aldebaran South 10 25	8	8	3	52	23	27	10	57	22
21	E	**4 Sun. Adv. St. Thomas**	8	8	3	52	23	28	morn.		23
22	M	[Shorteſt day	8	8	3	52	23	27	0	6	24
23	Tu		8	8	3	52	23	27	1	13	25
24	W		8	8	3	52	23	26	2	18	26
25	Th	Chriſtmas Day	8	7	3	53	23	24	3	24	27
26	F	St. Stephen	8	7	3	53	23	22	4	39	28
27	S	St. John	8	7	3	53	23	19	5	36	29
28	E	Sun. aft. Chriſtmas. In-	8	7	3	53	23	16	6	39	30
29	M	[nocents	8	6	3	54	23	13	☽ ſets		1
30	Tu		8	6	3	54	23	9	4	a 26	2
31	W	*Silveſter*	8	5	3	55	23	4	5	25	3

Days.	Leng. of Day.		Day decreaſe.		Day breaks.		Sun Eaſt.		Twilight ends.		Clock after Sun.		Seven Stars South.		
1	8	6	8	28	5	54	4	45	6	6	10	22	11	A	1
6	7	58	8	36	5	57	4	43	6	3	8	19	10		39
11	7	50	8	44	5	59	4	41	6	1	6	3	10		17
16	7	46	8	48	6	0	4	40	6	0	3	39	9		55
21	7	44	8	50	6	1	4	40	5	59	1	10	9		33
26	7	46	Incr.	2	6	0	4	40	6	0	1	20	9		10

ANSWERS *to the* QUERIES, REBUSSES, *&c. in Laſt Year's* DIARY

Query I. *Anſwered by* Caput Mortuum.

THIS difference is one of thoſe operations of Nature which, doubtleſs, will never be accounted for; though it is probably effected by attraction and repulſion; but in what manner?--- We obſerve that the *Sun-Flower* generally keeps turning its bloſſom towards the ſun; we behold with admiration, the phænomena of the *ſenſitive plant*, and *Venus fly-trap*, but when we would enquire the *cauſe* our reaſon is at a ſtand, and we are left to lament the circumſcribed ſtate of human knowledge.

Query III. *Anſwered by Mr.* I. Dalby.

Theſe ſeem to be the ſpecies of worms called by Linnæus, *Gordius aquaticus pallidus*, with black extremities; though I have ſeen ſome thouſands of them entirely black; but as he ſays they are bred in clay, it is probable that they change to a pale colour ſoon after coming into the water. Merrett, in his *Pinax Britannicarum*, calls them *ſeta aquaticus*, and mentions the ſame thing of their being vulgarly taken for animated horſe-hairs: his words are, " Vulgo creditur oriri, ex ſeta caudæ equinæ aquis immerſâ." He has not taken notice of their colour.

Query IV. *Anſwered by Mr.* French Johnſon.

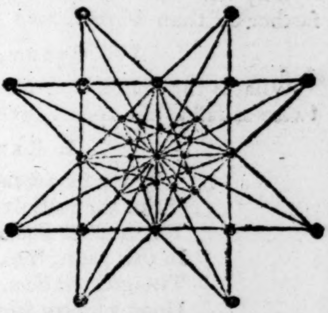
Sound the *s* in unlooſe ſoftly (as in looſe morals) and the myſtery will vaniſh; ſo then unlooſe morals will be good morals, and unlooſe will ſignify to be tyed.

Queries II. *and* V.

Are obliged to be deferred till next year, as no ſatisfactory anſwers have been received.

Anſwer to Mr. Dalby's *Paradox, with a new one propoſed, by Mr.* J Wales.

The ſcheme in the margin the muckle mon ſhows,
To plant three and thirty in twenty-four rows;
Now four in each row, and in rows that are even,
The number of plants, I would plant twenty-ſeven.



Anſwers to the three Rebuſſes, by Mr. John Clarke, *of* Lincoln.

I called laſt night on *Dalby*---he was gone
With *Rachel Rogers* up to *Iſlington*;
What could the errand be they went upon?

Anſwers to the Enigmas in laſt Year's Diary.

I. A Candle
II. A Woman
III. A Bed
IV. A Louſe
V. A piece of Muſic
VI. Coffee
VII. A Picture

An anſwer to all the Enigmas, by Mr. William Francis, *of* Reading.

Man! what is he? a reptile on the earth II.
A ſcene of miſery from his very birth;
His prime once paſt, how ſubject to decay!
Prone to the grave, and downward lies his way:
What various ills his every ſtate attend,
Each coming day but haſtens on his end;
To *bed* by ſickneſs, pain, and grief, confin'd, III.
All out of *tune* in body and in mind; V.
Vermin ſometimes, concomitants of age, IV.
Sadden the *picture*, and his death preſage; VII.
Teas, ſoops, and cordials, can't prolong his ſtay, VI.
But like a *candle* ſnuff he dies away. I.

Anſwer to the Prize, by Philomathes.

Your portraiture, ingenious *Clarke*,
Does elegant appear;
Pray write ſome more, but not too dark,
Againſt another year.

Ingenious Anſwers were alſo given by Meſſ. Rogers, Clarke, Dalby, G. Little, Johnſon, Moody, Wales, and ſeveral others; but the prize of ten Diaries fell to the lot of Mr. John Clarke.

New QUERIES, REBUSSES, *&c. to be anſwered next Year.*

I. QUERY, *by Mr.* Robert Moody.

What is the reaſon that dead bodies ſooner rot in a dry than a moiſt church yard?

II. QUERY, *by Miſs* Polly Tayrt.

Are not children naturally ambidextrous?

III. QUERY, *by Mr.* Iſaac Dalby.

Why does an object, when viewed with a magnifying lens, ſeem farther off than when viewed with the naked eye?

IV. QUERY, *by Mr.* John Burrow.

What is the reaſon that a body moving forward upon rollers, moves twice as faſt as the rollers themſelves?

I. REBUS, *by* Regulus.

If the faireſt fair you'd know,
Take the initials here below:
The higheſt ſtation and command,
In this great, free, and happy land;
The greateſt beauty or diſgrace
Upon a pretty female's face;
The point within the azure ſkies,
From whence the ſun is ſeen to riſe;
The city which ten years employ'd
The braveſt Greeks, before deſtroy'd.

II. REBUS, *by Mr.* Iſaac Dalby.

One third of the pleaſure of each toping blade,
When joined to a beaſt which the Lord never made,

Will tell you what brought an unfortunate bard,
To ample repentance in Lazarus' ward.

III. Rebus, *by Mr.* Iſaac Dalby.

A large purſe, and four ſevenths of a miſer,
With juſt the two-thirds of a ſheep;
Twice a letter of capital ſize, Sir,
Join'd to the beginning of ſleep;

Theſe name you a Sunday retreat,
Near London for cit and for ſtranger,
Where Venus and Mercury meet,
And your carcaſe and purſe are in danger.

Bag Nigg(and
e W(e) LL.
S

New Enigmas *to be anſwered in the next Year's* Diary.

I. Enigma, *by Mr.* William Francis, *of* Reading.

I Was born in a ſcuffle 'twixt father and mother,
And quickly convey'd to be nurs'd by another;
Tho' a black naſty jade, yet to tell you the truth,
She her duty perform'd, and befriended my youth:
A ſly beggar's brat thence ſtole me away,
And ſo altered my dreſs that I ſhine bright and gay;
I'm lively and briſk when I've food at command,
And chiefly ſubſiſt on the fat of the land;
On animal food tho' I moſtly do thrive,
I frequently feaſt on the ſpoils of the hive;
I'm always aſpiring, which haſtens my fate,
And my ruin compleats---a tale for the great:
Ye Enigmatiſts, who in dark myſteries delight,
In next Ladies Diary bring me to light.

II. Enigma, *by Mr.* T. Fiſhbourne.

Ye peaceful bards a-while attend,
And hearken to a faithful friend;
A friend you'll ſay, I make no doubt,
When once my name you have found out.
My downy wings around me ſpread,
My healing balm propitious ſhed,
Exert my kind relieving art,
And heal the ſorrow wounded heart;
I am a kind conſoling gueſt,
And calm the tumults of your breaſt;
I gently ſooth your ſoul to peace,
And make each jarring paſſion ceaſe;
From me your chiefeſt bleſſings flow,
A cordial I'm for every woe;
I chear your gloom, to joys invite.
And make your cares and burdens light;
From envy, pride, and diſcord free,
Are every one poſſeſſed of me,

All ſeek me in a different way,
Then what's my name, ye witty ſay.

III. ENIGMA.

Who's he that's no bodys friend,
Whoſe levees yet great men attend;
Who in retirement loves to ſneak,
Yet for domeſticks, oft does ſeek?
Folly and innocence him dread,
He's hated, yet he's follow'd,
And is interr'd before he's dead.
His retinue's kept at others coſt,
And when he's curſt he proſpers moſt.

IV. ENIGMA.

I ſtand but on one leg, yet do ſuſtain
Much weight, beſide a noted rogue in grain,
And 'twere an ill wind which blew him no gain.
He gives me clothes when faſt he'd have me run,
But ſtrips me naked when his work I've done;
Then I, with arms acroſs, expos'd do ſtand,
Forc'd to ſubmit to every turn of hand,
And to inconſtant unſeen powers command.
I once encounter'd was by hardy fool,
Who'ad got my nameſake lodg'd within his ſcull;
He me attack'd in wild and frantic mood,
And I my ground, tho' in ſwift motion, ſtood;
He from my arms receiv'd a ſtunning blow,
Yet what I was the coxcomb did not know;
And you're more wiſe, If you gueſs what I'm now.

V. ENIGMA.

Cloſe to my owner I adher'd,
'Till bloody hands me from him tear'd;
In warmth and quietneſs we liv'd,
And, while together, well we thriv'd;
But naked now men me expoſe,
And I excite them too to blows.
Dumb was I born, ſtill have no voice;
Yet courts and camps I fill with noiſe.
I liv'd in peace, now ſerve in wars,
Was innocent, but now at bars
Am try'd, where I move endleſs jars.
Great rogues trade in me by whole-ſale,
In parcels too they me retail;
But when their greater uſe I fail,
Small louſy thieves do in me deal,
And ſerve their ends of me piece-meal.

PRIZE ENIGMA *(of 10 Diaries) by Mr.* Iſaac Dalby.

Ye meddlers, who are always rude,
And unpolitely will intrude

Like Marplot, and cannot forbear
To thruſt your noſes every where,
Be circumſpect---I'm one in keeping,
That pays impertinence for peeping----
Not care I, tho' perhaps in huff,
You take at once diſguſt and ſnuff.
There's ne'er a Slakenbergius-ſnout,
Nor Proclus' like, ſo large about,
That poets ſing, he could not wipe it,
His hand b'ing much too ſmall to gripe it;
Nor pimpl'd knob, nor that with ſcars,
Curtail'd of half in Venus' wars,
That I reſpect,---for great and ſmall,
I play St. Dunſtan with 'em all.---
And this is done, Sirs, in a trice,
Tho' I'm not ſhap'd like tongs or vice;
But rather ſeem, (except in colour)
Like Mynheer Van Dunk's Kevenhuller;
With mouth extenſive, deep and round,
Deſcending to a depth profound---
Yet like a hag, long paſt her prime,
Whoſe teeth are drawn by quacks and time,
I am, tho' odd is the relation,
Incapable of maſtication;
But each fair belle by kindneſs led,
Prepares my food before I'm fed,
Then after, which you'll think is aukward,
They take great pains to feed me backward---
Laborious taſk! which brings to view
Things ſeldom ſeen, or ſeen by few;
But this alas! diſturbs my reſt,
And ſtorms invade my peaceful breaſt---
Loud thunders roll, and winds long pent,
In caverns deep now find a vent;
Rocks burſt, and with impetuous ſweep,
Are hurl'd into the briny deep.
From yon black cloud which ſeems to rend
In twain, the rattling ſtreams deſcend,
Waves upon waves now ſeem to ride
And iſlands float along the tide;
While dreadful as a cataract roars,
The ſurges 'gainſt the neighb'ring ſhores---
But ſtraight there ruſhes from behind,
Some poet damn'd, to me conſign'd,
Who gently on the ſurface glides,
And then the raging ſtorm ſubſides.
  Now ladies, after this diſgrace,
Dare you to look me in the face?---
No:---and tho' daily I befriend ye,
'Tis ten to one but I offend ye.
  Not fam'd at all for much diſcerning,
I cannot boaſt of taſte or learning;

A
C . . . e
S . . L

Yet of what's form'd by nature's hand,
The fundamentals underſtand;
My aid ſubſervient to her laws,
Is ſought when ſhe'd her paths diſcloſe;
Behold an Æſculapian big,
With cane and large important wig,
And pair of ſupplemental eyes,
(The certain marks of being wiſe)
Explore my bowels for the ſtate,
Of health and ſearch for hidden fate---
In vain,---no ſecrets with me reſt,
Tho' daily lodg'd within my breaſt.
  Know I'm compared to a punk,
But never was detected drunk;
Yet in North Britain, as 'tis ſaid,
I puke upon each ſtranger's head,
A moſt uncivil ſalutation,
Tho' not peculiar to that nation,
For the Athenian Sage of old,
The ſame experienc'd from a ſcold.
Now ſhould you ever me aſſail,
I'll make your worſhip turn your tail,
And tho' you'd ſtop me you will find,
That fearleſs I am cloſe behind.

Anſwers to the Mathematical Queſtions *propoſed in laſt Year's* DIARY.

I. QUESTION, *anſwered by Mr.* Robert Moody.

IT is evident that if B advances his goods $13\frac{1}{2}$ per cent. and allows $7\frac{1}{2}$ per cent. advance on A's ſugar for paying $\frac{1}{4}$ of the amount in ready money, that the whole of A's advance muſt be 21 per cent. then $121 \times 6,25 \div 100 =$ the price of 1 lb. and 24480 pence, the price of 36 pieces of B's goods, divided by the price of 1 lb. is $3237\frac{3}{121}$ the number of pounds of ſugar; and 2*l.* 16*s.* 8*d.* × 12 = 34*l.* the ready caſh which A gives B for his ſugar.

II. QUESTION *anſwered by Lieutenant* Wheldale.

Analyſis. Let AB the baſe, ACB a ſegment of a circle containing the given vertical angle, and ACB the required triangle, draw $FZ \perp$ to FK and the perpendicular CZ upon it, then by a known property $AK + KB : AC + CB :: \sqrt{KF} : \sqrt{CZ}$, therefore $AC + CB = 2AK\sqrt{CZ} \div \sqrt{KF}$, wherefore S or $AC + CB + CD = 2AK\sqrt{CZ} \div \sqrt{KF} + CD = S$, let $4AK^2 \div KF = R$, then $S - CD = \sqrt{R \times CZ} = \sqrt{R \times CD + R \times DZ}$, conſequently $S^2 - R \times FE = \overline{R + 2S} \times DC - DC^2$, whence this conſtruction. Take $EQ = R + 2S$ and cut it in n ſo that $Qn \times nE = S^2$

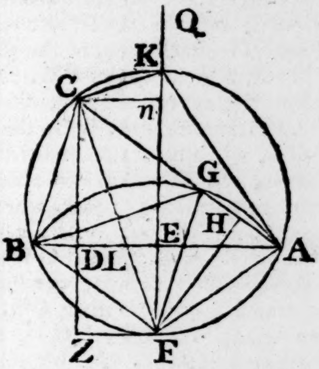
$= S^2 - R \times FE$ and draw nC ∥ to AB, cutting the circle in C, the vertex of the required triangle.

Note. This is prob. 5 of Newton's Univerſal Arithmetick.

The ſame anſwered by Mr. Jeremiah Ainſworth.

Construction.

Having drawn the circle, &c. as before, take $EQ =$ the ſum of the ſides and perpendicular, draw alſo AK and to twice AK let a line be added ſo that the rectangle of the part added, and the whole be $= FQ \times FK$, then apply the chord FC equal to the additional part, and join A, C, and B, which will be the triangle required.



For from F with the diſtance FA or FB deſcribe a circle, let fall the perpendiculars CD and FH, and join the lines as in the figure, then $CF \times FL = FE \times FK$ and $CD \times KF = CL \times CF$ by the known properties of the circle, but $CL \times CF = CF^2 - CFL = CF^2 - KFE$, alſo from the ſimilar triangles CFH and KFA, $CH \times KF = CF \times KA$, whence it follows that $2CH \times KF + CD \times KF$, or $\overline{2CH + CD} \times KF = 2KA \times CF + CF^2 - KFE$; and conſequently $\overline{2CH + CD + FE} \times KF = \overline{2KA + CF} \times CF$, which is, by conſtruction, equal to $FQ \times FK$, wherefore $2CH + CD + FE = FQ$ and $2CH + CD = EQ$; but $2CH = CB + CA$ by *prop.* 9. *article* XII. wherefore $2CH + CD = EQ = CB + CA + CD$ - - - Q. E. D.

Limitation. EQ muſt not be greater than $2AK + KE$.

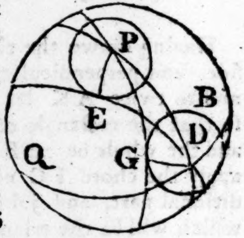
III. Question.

A ſmall omiſſion was made in copying this queſtion for the preſs; however, as that which the propoſer intended, may be eaſily reſolved by Prob. III. Art. 9, in laſt year's Diary, as well as moſt other problems of the ſame kind, wherein the limits of the ſum or difference of the ſides are concerned, thoſe queſtions ſeem to require no other notice than a reference to the aforeſaid article.

IV. Question, *anſwered by Mr.* J. Ainſworth.

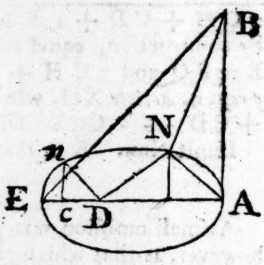
By prop. 22, Simpſon's Trig. as cot. of half the obliquity of the ecliptic is to its tangent, ſo is the ſine of the ſum of the ſun's longitude and right aſcenſion, to the ſine of their difference; hence when the difference is a maximum the ſine of the ſum will be ſo too, and conſequently equal to radius, and the ſum itſelf $=90$ degrees, whence the difference will be $2^\circ.\ 28'$. and the longitude $= 46^\circ.\ 14'$. which anſwers to May 7th; and it is evident that the common increaſe of longitude and right aſcenſion during the interval, muſt be 180 degrees, whence the time will appear to be November 8, and the interval 185 days, conſequently the principal will be 118*l.* 7*s.* 6*d* ¾. *Solution were alſo given by Meſſ. Aſpland, Barker, Boucher, Fininley, Hardy, Lynn, and Moody.*

V. QUESTION, *anſwered by Mr.* Isaac Dalby, *the propoſer.*



Let P be the given point, and QB the great circle. Through P draw a great circle PD at right angles to QB, then about the points P, D, as poles, deſcribe two leſſer circles, ſo that their diſtances are each equal to the given leg, through P, D draw great circles PG, DE, to touch the leſſer circles reſpectively, then having drawn the great circles PE, DG, the triangles DEP, PGD, will anſwer the conditions of the problem; that is, the ſide DE is a *min.* and its complement to a ſemicirc. a *max.* when the given ſide PE is drawn from the given point P; but if the given ſide DG (PE) ſalls into the given great circle QB, then PG is a *min.* and its complement to a ſemi circ. a *max.*---For PD being the ſhorteſt portion of a great circle that can be drawn from P to meet QB, and the hypothenuſe common to both the triangles DEP, PGD, therefore DE, PG are each a *min.* and their complements to ſemicircles, forming two other right-angled triangles, muſt be each a *max.*

VI. QUESTION, *anſwered by Mr.* Vidgen, *of the* Tower, London.



Let AB $= b$, length of the ſtring BAD $= m$, AD $= n$, AC $= x$, then DC $= n \backsim x$ and let CN $= y$, BN^2 $=$ AB^2 $+$ AC^2 $+$ CN^2 $=$ AB^2 $+$ AC^2 $+$ ND^2 $-$ DC^2 $= \overline{m - \mathrm{ND}}|^2 = m^2 - 2m \times$ ND $+$ ND^2, whence $m^2 - 2m \times$ ND $=$ AB^2 $+$ AC^2 $-$ CD^2, that is, $m^2 - 2m \sqrt{n^2 - 2nx + x^2 + y^2} = b^2 - n^2 + 2nx$. But from this equation to determine the nature of the curve, let $zn = m$ then will $b + n = zn$ and $b = \overline{z - 1} \times n$, and we ſhall have $z^2 n^2 - \overline{z^2 + 2z - 1} \times n^2 + n^2 - 2nx = 2zn \times \sqrt{n^2 - 2nx + x^2 + y^2}$, $\overline{zn - x}^2 = z^2 \times \overline{n^2 - 2nx + x^2 + y^2}$, $\frac{x^2}{z^2} - \frac{2nx}{z} = x^2 - 2nx + y^2$ and $\frac{z^2 - 1}{z^2} \times x^2 - \frac{2z - 2}{z} \times nx + y^2 = 0$. Let the tranſverſe diameter EA $= a$ then BE $= \sqrt{a^2 + \overline{nz - n}^2}$, and BE $+$ ED $= \sqrt{a^2 + \overline{nz - n}^2} + a - n = zn$, whence by reduction $n = \frac{\overline{z + 1} \times a}{2z}$, which being ſubſtituted we have

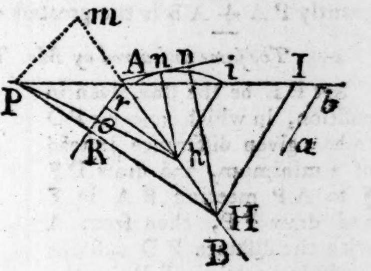
$$y^2 = \frac{z^2 - 1}{z^2} \times \overline{ax - x^2}$$ the equation for an ellipſis.

A very neat and general ſolution was alſo given by Mr. Brown, the propoſer.

The ſame anſwered by Mr. Jeremiah Ainſworth *of* Mancheſter.

It is evident that if a plane be ſuppoſed to paſs through the given points B and D, a conic ſection will be deſcribed thereon by a point keeping the cord tight, whether the ſum of the parts of the cord be given (as in the queſtion) or their difference, and it will be an ellipſe in the firſt caſe, an hyperbola in the ſecond, and a parabola when one of the points is ſuppoſed to be removed to an infinite diſtance; now if this plane, with the figure thereon, be revolved about the line B D, a ſolid will be generated by the curve, the interſection of which by any plane whatever, it is known will be a conic ſection; and, therefore, whatever angle the planes in the queſtion are ſuppoſed to make, the curve will be an ellipſe, except when one of them is perpendicular to the line B D, in which caſe it will be a circle. *In a manner very little different the ſolution was given by Mr. Iſaac Dalby and Mr. John Burrow.*

VII. QUESTION, *anſwered by Mr.* Iſaac Dalby.

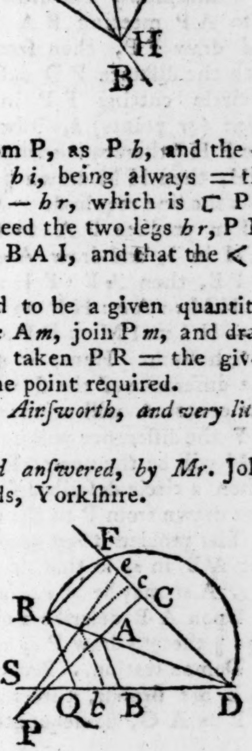


Conſtruction. From any point in A H as b, draw a line $b\,i \parallel b\,a$ (the line given in poſition) with which as radius deſcribe an arc, $i\,n$, from A draw a tang. thereto, and make the $\angle$ R A B $=$ $\angle$ B A n, from P draw P H $\perp$ A R, and H I $\parallel$ $b\,a$, and the thing is done. For drawing H n, $b\,n$ $\perp$ A n, we have by ſim. $\triangle$, s, H n $=$ H R $=$ H I, therefore P H $-$ H I $=$ P R, which is a minimum, becauſe if any other line be drawn from P, as P b, and the $\perp$ $b\,r$ let fall upon A R, then the lines H I, $b\,i$, being always $=$ the perp. H R, $b\,r$; therefore P b $-$ $b\,i$ $=$ P b $-$ $b\,r$, which is $\sqsubset$ P R by what the two hypothenuſes $b\,o$, P o exceed the two legs $b\,r$, P R. Here it is neceſſary that the $\angle$ A $b\,a$ $\sqsubset$ $\angle$ B A I, and that the $\angle$ P A R, B A I be leſs than right ones.

If inſtead of a *min.* the diff. was required to be a given quantity, produce B A till $b\,i$: b A : : the given diff. : A m, join P m, and draw A R parallel thereto, then from P having taken P R $=$ the given diff. and produced it to meet A B, it gives the point required.

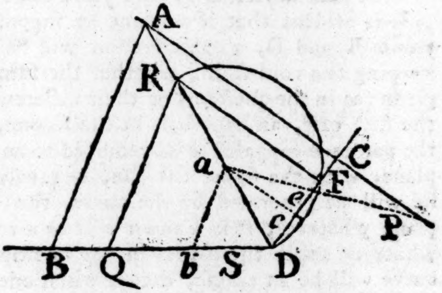
A ſolution equally elegant was given by Mr. Ainſworth, and very little different from the following one.

The ſame problem rendered more general, and anſwered, by Mr. John Burrow, of Rounday, *near* Leeds, Yorkſhire.



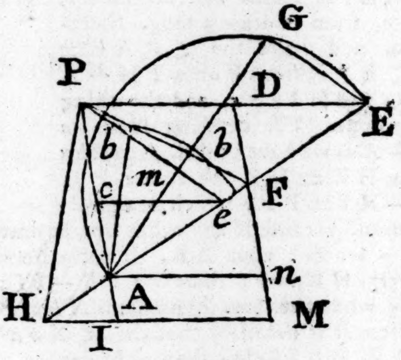
Let D R, D S be two lines given in poſition, P a given point, R Q a line given in poſition; it is required to draw P A to cut D R in A, ſo that drawing A B parallel to R Q cutting D S in B, the ſum or difference of P A and A B may be the leaſt poſſible.

On any line D R deſcribe a ſemicircle, in which let R F be inſcribed equal to R Q; join D F, and draw P C $\perp$ to D F



cutting DR in A, then if AB be drawn parallel to RQ, PA and AB are the lines required. For draw any other line Pa and $ab \parallel$ to AB, alſo draw $ac \parallel$ to AC; then becauſe $\left\{\begin{matrix} Pa + ac \\ Pa - ac \end{matrix}\right\}$ is $\left\{\begin{matrix} \sqsubset \\ \sqsupset \end{matrix}\right\}$ than $\left\{\begin{matrix} PA + AC \\ PA - AC \end{matrix}\right\}$ and $Pa \pm ac = Pa \pm ab$, therefore $Pa \pm ab$ is $\left\{\begin{matrix} \sqsubset \\ \sqsupset \end{matrix}\right\}$ than $PA \pm AB$, or $PA \pm AC$; for $AC = AB$ becauſe $RQ = RF$, conſequently $PA \pm AB$ is the greateſt or leaſt poſſible. Q. E. D.

The ſame anſwered by Mr. Thomas Moſs, *the propoſer.*



Let PE be the line given in poſition, in which conceive PD to be a given difference inſtead of a minimum, and draw $DF \parallel$ to AP meeting BA in F and draw FP, then from A with the diſtance PD deſcribe a circle cutting FP in the point (or points) b, draw Ab and $PH \parallel$ thereto meeting AB in H, then HM drawn $\parallel$ to EP is the line required.

For produce PA to meet HM in I, and draw $An \parallel$ to to PE, then $AF : FH :: An : HM :: Ab : HP$ and becauſe $An = Ab$, $HM = HP$ and $\therefore HP - HI = IM = PD$.

Scholium. Hence it appears that the problem is impoſſible when the difference of the ſides is ſuch that a circle deſcribed therewith from the center A will neither cut nor touch PF, and that when it touches PF the difference will be the leaſt, for it may be eaſily proved that DFM will be the neareſt line that can be drawn $\parallel$ to AP meeting AE, when a circle deſcribed from A (as above) does not cut but touch the line drawn from P to the interſection in AE.

The problem then becomes this---From P to draw a line Pe meeting AE in e, ſo that ec being drawn $\parallel$ to EP and Am perpend. to Pe, Am may be $= ec$, and the conſtruction is as follows:

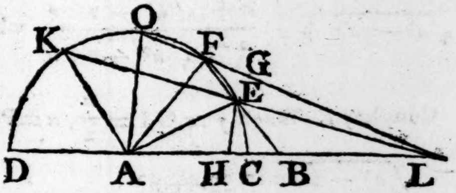
Upon AE deſcribe a circle, in which apply $AG = EP$; draw EG, and $\parallel$ thereto draw Pe, and the thing is done.

Demonſtration. Becauſe the triangles Ace, APE, Ame and AGE are ſimilar, therefore $Ae : AE :: ec : EP :: Am : AG$, but $EP = AG$, conſequently $ec = Am$.

VIII. Question

VIII. QUESTION, *anſwered by the Rev. Mr.* Lawſon, *the propoſer.*

We muſt firſt take notice that the firſt member of this queſtion was wrong printed. Inſtead of the *ratio of the angle* A O L *to* A K L, it ſhould have been *the ratio of the arc* F C *to* E C.



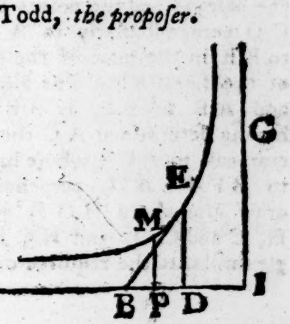
1. Now the ratio of the arc F C to E C is thus ſhewn to be greater than the ratio of the angle F L C to E L C. From the center A draw A F, A E. draw the chord F E, which produced may meet the diameter in B. and with L center and radius L E deſcribe the arc H G. Sector A F E : △ A E B is greater than △ A F E : △ A E B. But Sect. A F E : Sector A E C is greater ſtill. ∴ Sect. A F E : A E D, i. e. arc F E : E C is greater than △ A F E : △ A E B, i. e. than line F E : E B. and by inverſion arc E C : arc F E is leſs than line E B : line F E. Juſt in the ſame manner we may ſhew that Sect. H L E : Sect. E L G, i. e. arc H E : arc E G is greater than △ B L E : △ E L F, i. e. than line B E : line E F. ∴ B E : E F is leſs than arc H E : arc E G. Since then arc E C : arc F E is leſs than line E B : line F E, and line E B : line F E is leſs than arc H E : arc E G, ∴ arc E C : arc F F is leſs than arc H E : arc E G, i. e. than angle A L E : angle E L F. ∴ by perm. and comp. arc F C : to arc E C is greater than angle F L C : angle E L C. Q. E. D.

2. Let A O, A K be joined. Then by the 1ſt part and permutation, angle F A L : ang. F L A is greater than ang. E A L : ang. E L A, and by comp. F A L + F L A or A F O or A O F : F L A is greater than E A L + E L A or A E K or A K E : E L A, that is, A O L : O L A is greater than A K L : K L A, or by perm. A O L : A K L is greater than O L A : K L A. Q. E. D.

3. Since by part 2d and permutation A O L : O L A is greater than A K L : K L A, by comp. A O L + O L A or D A O : O L A is greater than A K L + K L A or D A K : K L A, and by perm. D A O : D A K : : arc O D : K D is greater than ang. O L A : ang. K L A.

This queſtion was alſo anſwered by Mr. Iſaac Dalby and Mr. John Burrow. N. B. The method of reſolving the 9th queſtion is ſelf-evident from the 7th prop. of art. ix. in laſt year's Diary.

X. QUESTION, *anſwered by Mr.* Todd, *the propoſer.*



If $a = 400$, $n = $ P M, P D $= x$, and D E $= y$, then $\dot{y} : \dot{x} : : y : \frac{y\dot{x}}{\dot{y}} =$ D B; alſo the tangent B E $= \frac{y}{\dot{y}} \sqrt{\dot{x}^2 + \dot{y}^2}$ and $y + \frac{y}{\dot{y}} \sqrt{\dot{x}^2 + \dot{y}^2} = a$ by the queſtion, whence $\dot{x} = \frac{\dot{y}\sqrt{a^2 - 2ay}}{y}$, and thence the equation of the fluents is $x = 2\sqrt{a^2 - 2ay} - a\,|\,\dfrac{a + \sqrt{a^2 - 2ay}}{a - \sqrt{a^2 - 2ay}} -$ $\sqrt{a^2 - 2an} + a\,|\,\dfrac{a + \sqrt{a^2 - 2an}}{a - \sqrt{a^2 - 2an}}$, where $x = o$ when $y = n$.

Corollary 1. When $y = GI = \dfrac{a}{2}$, $x = PI = a\,|\,\dfrac{a + \sqrt{a^2 - 2an}}{a - \sqrt{a^2 - 2an}}$ $- \sqrt{a^2 - 2an} = 39{,}44492$.

To find the curve $ME = z$; becauſe $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2} = \dfrac{a\dot{y}}{y} - \dot{y}$, we have $z = a\,|\,y - y - a\,|\,n + n = a\,|\,\dfrac{y}{n} - y + n = ME$.

Corollary 2. When $y = GI$, $z = MEG = a\,|\,\dfrac{a}{2n} - \frac{1}{2}a + n =$ $65{,}0728$.

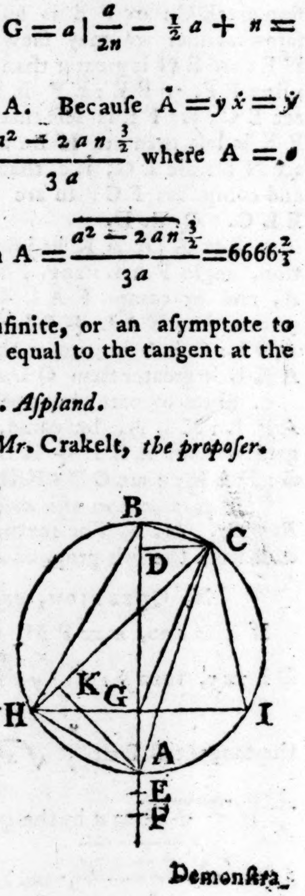
Laſtly, to find the area of $PMED = A$. Becauſe $\dot{A} = y\dot{x} = \dot{y}$ $\sqrt{a^2 - 2ay}$, $A = -\dfrac{\overline{a^2 - 2ay}|^{\frac{3}{2}}}{3a} + \dfrac{\overline{a^2 - 2an}|^{\frac{3}{2}}}{3a}$ where $A = 0$ when $y = n$.

Corollary 3. When $y = GI = \frac{1}{2}$, then $A = \dfrac{\overline{a^2 - 2an}|^{\frac{3}{2}}}{3a} = 6666\frac{2}{3}$ the area of PMGI.

Scholium. When $y = c$, x will be infinite, or an aſymptote to the curve, and the greateſt ordinate GI is equal to the tangent at the point G.

This queſtion was alſo anſwered by Mr. J. Aſpland.

XI. QUESTION, *anſwered by the Rev. Mr.* Crakelt, *the propoſer.*



Conſtruction. Upon any aſſumed line, AB, as diameter, deſcribe a circle; and, having formed the angle BAC equal to half the given difference of the angles above the baſe, joined the points B, C, and drawn CD perpendicularly to AB, make 2 BC to BE in the ratio of the given difference of the ſides to the line biſecting the baſe, and AB to BE, as BE to BF; then having determined AG the leſs of two reciprocals to AC^2, whoſe ſum may be equal to $BF + 2AD$, perpendicularly to AB draw the chord HGI, join the points H, C and C, I, and HCI will be a triangle ſimilar to the required one.

Demonſtration. Draw C G, A H, and A K perpendicularly to H C. Then, ſince by *con.* $AG \times BF + AG \times 2AD - AG^2 = AC^2 = CG^2 + AG^2 + AG \times 2DG$ (*Euc.* ii. 12.) $= CG^2 + AG^2 + AG \times \overline{2AD - 2AG} = CG^2 + AG \times 2AD - AG^2$, therefore will $CG^2 = AG \times BF$. But, by ſimilar triangles $HK^2 : BC^2 :: AH^2 = AB \times AG$ (*Euc.* vi. 8. cor.) $: AB^2 :: AG : AB$ (*Euc.* vi. 1.) $:: AG \times BF = CG^2 : AB \times BF = BE^2$ by conſtruction; conſequently, by permutation, $HK^2 : CG^2 :: BC^2 : BE^2$, or $HK : CG :: BC : BE$. Now it is well known that H K is equal to half the difference betwixt H G and L I, wherefore by doubling the antecedents of the laſt proportion, we ſhall have, $2HK$ or $HC - CI : CG :: 2BC : BE$. And that the difference betwixt the angles C I H and C H I is equal to 2 B A C is manifeſt; becauſe the difference betwixt the arches H C and I C is equal to twice the arch B C.

Scholium. If with the other data, the ſum inſtead of the difference of the ſides had been given, make 2 A C to B E in the ratio of the ſum of the ſides to the biſecting line, and CG^2 equal to $BG \times BF$, that is, B G the leſs of two reciprocals to BC^2 whoſe ſum is $BF + 2BD$, and in the demonſtration uſe C K, A C, and B H inſtead of H K, B C, and A H, and every thing elſe will follow.

Very elegant ſolutions were alſo given by Mr. George Sanderſon, and Mr. Iſaac Dalby; Mr. Ainſworth alſo gave excellent ſolutions both to the queſtion itſelf, and that mentioned in the above ſcholium, with ſeveral others, ſome of which will be inſerted in future.

XII. Question, *anſwered by* Archimedes.

Suppoſe A, B, C, and D to be the four players, A being the dealer, then by prop. 6. corollary 2, of Simpſon's Chances, the probability that any one of the players B, C, D, has of holding not more than four trumps will be expreſſed by $\frac{39 \cdot 38 \cdot 37}{51 \cdot 50 \cdot 49}(12) \times \left[\frac{27}{39} + \frac{1}{39} \times 13\ 12 + \frac{1}{28 \cdot 39} \times 12 \cdot 13\ \frac{12}{1}\ \frac{11}{2} + \frac{1}{29 \cdot 28 \cdot 39} \times 11 \cdot 12 \cdot 13\ \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3} + \frac{1}{30 \cdot 29 \cdot 28 \cdot 39} \times 10 \cdot 11 \cdot 12 \cdot 13 \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3}\ \frac{9}{4}\right]$ which reduced is $= \frac{432385952}{466921735}$, which taken from unity there will remain $\frac{64535783}{466921735}$ for the probability that each of the players B, C, D, has of holding 5 or more trumps, and from the ſame problem it is evident that the probability of the dealer's not holding more than four trumps will be expreſſed by $\frac{39 \cdot 38 \cdot 37}{51 \cdot 50 \cdot 49}(12) \times \left[1 + \frac{1}{28} \times 12 \frac{12}{1} \times \frac{1}{29 \cdot 28} \times 11 \cdot 12\ \frac{12}{1}\ \frac{11}{2} + \frac{1}{30 \cdot 29 \cdot 28} \times 10 \cdot 11 \cdot 12 \frac{12}{1}\ \frac{11}{2}\ \frac{10}{3}\right]$ which reduced will be =

$\frac{331188221}{466921735}$, and therefore $\frac{135733514}{466921735}$ will be the probability that the dealer A holds 5 or more trumps; conſequently $3 \times \frac{64535783}{466921735} + \frac{135733514}{466921735}$ or $\frac{329340863}{466921735}$ will at laſt expreſs the probability that ſome one of the four players holds 5 or more trumps; and therefore the required odds that ſome one of the players holds 5 or more trumps are as 329340863 to 137580172, being nearly as 12 to 5, or ſtill nearer as 67 to 28.

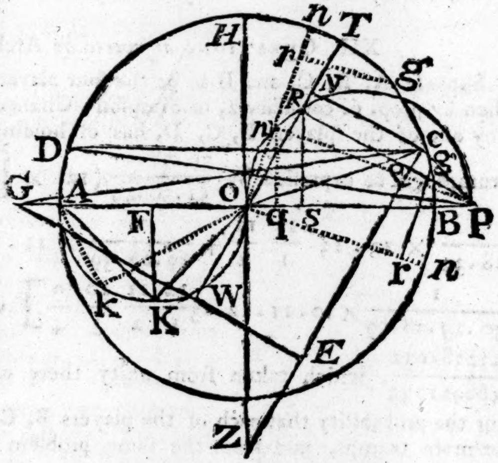
Note, the probability of ſome two of the players each holding 5 or more trumps, being inconſiderable, is neglected.

Nearly in the ſame manner this queſtion was alſo anſwered by Mr. Robert Moody, and Mr. Ainſworth, &c.

Prize Question, *anſwered by Mr.* Iſaac Dalby.

Lemma. If upon a given hypothenuſe A O a right angled triangle A K O be conſtructed, the rectangle of the legs $AK \times OK$ will be a maximum when they are equal.----For letting fall the $\perp$ F K from the cent. F, and let the ſemicirc. A K O be deſcribed, then will $HK = OK$, and by ſim. $\triangle s$ we have $AK \times OK = AO \times FK$ which is a max. becauſe A O is conſtant, and F K the greateſt $\perp$ to A O that can be drawn within the ſemicirc.

Conſtruction. Let A B be the diam. O the center, and P the given point. Upon O P, A O, let ſemicircles be deſcribed, take O K, A K equal to each other and in the ſemicircle O P and $\perp$ O P make R S a fourth proportional to P O, O K, A K, through R draw O T, in which take $ON = OK\ (AK)$ and draw $NC \perp NO$ meeting the circumf. in C, then draw the chord $CD \parallel AB$ and the thing is done.



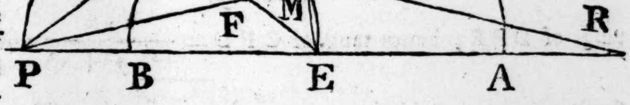
Demonſtration. Join D P, P C, P R, O C, produce B A till $AG = BP$ and draw C G, alſo let O H be $\perp$ A B, and C Z be drawn $\parallel$ N O meeting H O produced in Z, alſo draw $GE \perp CZ$, and produce N O to W.

Since by conſtruction ON (OK, AK) is a mean proportional between OP, RS, and the $\angle ORP$ a right one, it is alſo a mean proportional between OR, RP, that is, $OR : ON :: CN : RP$ (becauſe $OA = OC$, and $ON = OK = AK$, CN is $= ON$) whence by compoſition and diviſion $ON + OR : ON - OR :: RP + CN : RP - CN$; but becauſe GE, PR, CN are $\parallel$ to each other and $\perp CZ$, NW, we have $ON + OR = CE$ (OW being $= OR$) $ON - OR = CQ$, $RP + CN = GE$ and $RP - CN = PQ$, hence the laſt proportion becomes $CE : CQ :: GE : PQ$, therefore the $\triangle s$ GCE, PCQ are ſim. and ſo CZ biſects the $\angle GCP$, hence if a circle is conceived to paſs through the points P, C, D, G, it will alſo paſs thro' Z, and the $\angle CZH$ will be $= \frac{1}{2}$ the $\angle CPD$, but the $\angle CZH = TOH = RPO$; now the $\angle RPO$ is evidently a max. when RS is a max. or when $PO \times RS$ is a max. but $PO \times RS = NO \times CN$ (by conſtruction) which (becauſe $NO = CN$) is a max. by the foregoing lemma.

If it is required that the $\angle CPD$ ſhall be of a given mag. inſtead of a maximum, the conſtruc. will be thus.——Draw On, On making the $\angle s$, HOn, BOn, each $=$ half the propoſed $\angle$, draw $Pw \perp On$ and let fall the perp. wq, then in the ſemicircle AO having taken Ok, Ak, ſo that their rectang. may be $= OP \times wq$, make Or, Or each $= Ok$, and draw the perpendiculars rg, rg, then if chords be drawn from the points g, $g \parallel AB$, either will anſwer the conditions of the prob.—The demonſtration is evident from that already given.

The ſame anſwered by Reuben Burrow, *the propoſer.*

Analysis. Suppoſe the thing done, and let C be the required point, P the point in the diameter produced, and CD the chord required; alſo let $ER = EP$, (E being the center) and join the points D, C, P, and R; then it is evident that CPD is the difference of the angles CPR and CRP, but CPD being a maximum, this difference is a maximum alſo; Now when the difference of CPR and CRP is conſtant, it is well known that the vertex C is an hyperbola, paſſing thro' P; therefore when this difference is the greateſt, it is evident the hyperbola will touch the circle in the point C; and if ES be ſuppoſed one of the aſſymptotes paſſing thro' the center E, and CS, PM perpendicular thereto, $CS \times SE$ will be equal to $PM \times ME$; but becauſe every other point of the hyperbola, except C, falls without the circle

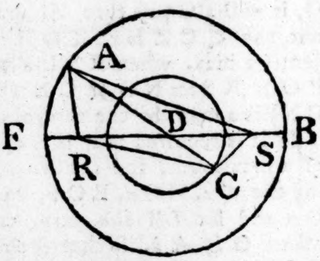


circle, it is evident that C S × S E muſt be a maximum; but C E being given, and C S E a right angled triangle, C S × S E will be greateſt when C S = S E, wherefore the triangle C S E or its equal P M E is given; whence this *Conſtruction.*

On P E deſcribe a ſemicircle, inſcribe therein the triangle P M E whoſe area is half the ſquare of the radius of the given circle; take M F = M E and E F being joined will cut the circle in C the point required.

A different ſolution may be deduced from the 59th problem of Simpſon's Algebra; but the problem will be conſidered in a more general manner ſome future opportunity.

Corollary. Hence if B A F be a given circle, and the points R, S in B F equidiſtant from the center D, the point A may be found, where the difference of the angles R A D, D A S is the greateſt; for take D C a third proportional to D F and D R, and with that diſtance and the center D, deſcribe a circle, then find the point C by the foregoing problem where the difference of R C S and C R S is the greateſt, and C D produced to cut the other circle gives the point A required. For $AD \times DC = DR^2 = DR \times DS$ ∴ A, R, S and C are points in a circle, conſequently S A D = S R C and R A D = R S C, therefore R A D − D A S is the greateſt poſſible.



Scholium. The problem in the laſt corollary has been thought worthy of the attention of ſeveral learned men, particularly the famous *P. Friſi*, who in the *Atta del' Siena* has beſtowed ſeveral pages thereon; the concluſion there given is exceeding ſimple, but the proceſs is in effect fluxional; *Cramer* has alſo given a fluxional ſolution in his "*Analyſe des Lignes Courbes*," but as this problem is nothing more than a corollary to the laſt, and as I have received anſwers by fluxions to it from a great many ingenious correſpondents, I ſhall inſert one of them, eſpecially as no leſs than twelve different people have ſolved it almoſt exactly the ſame way, viz. *Meſſrs. J. Aſpland, Edward Boucher, D. Cunningham, W. Dixon, W. Fininley, W. Francis, W. Hardy, J. Hartley, James Pringle, John Roper, Thomas Todd, William Wilkin.*

Let $BE = a$, $PE = B$, and the perpendiculars C N and $DW = x$, alſo let $EN = EW = y$, then $\frac{x}{b-y}$ = tang. of C P E and $\frac{x}{b+y}$ = tang. of D P E; hence tang. of C P D $= \frac{2yx}{b^2 - y^2 + x^2}$ which put into fluxions and reduced, gives $x = \frac{a}{b}\sqrt{\frac{b^2 - a^2}{2}}$ the diſtance of the chord required from A B.

A geometrical ſolution was alſo given by Mr. Ainſworth, who is entitled to the prize of twelve Diaries, the ſilver medal was adjudged to

Mr. Iſaac Dalby, as his ſolution was the only geometrical one that came in the limited time.

ARTICLE XI.

A Supplement to a former Article, concerning the Equation of Payments. by Reuben Burrow.

As there is ſcarcely any ſubject that has cauſed more diſputing and wrangling among arithmetical writers, than the equation of payments; and as the lateſt writers on arithmetic have only given us the miſtakes of former authors intermixt with peremptory aſſertions and invidious remarks of their own, I thought it might be a means of putting a ſtop to ſuch reflections by conſidering the ſubject in a more general manner than it has hitherto been. In order to this, let us ſuppoſe that one perſon owes another the ſums of money M, N, P, and Q, &c. payable at different times, and that the creditor is willing to receive the whole ſum at one ſingle payment, at a time when it will be of equal advantage to him whether he receives it thus, or receives the payments in their proper order; let us, in the firſt place, ſuppoſe the creditor to receive his debts as they become due, then it is evident that at the time of the laſt payment he will have received the ſum of the ſingle payments, together with the intereſt ariſing from each, from the time of becoming due to the time of the laſt payment; and it is alſo evident that if the debtor had paid the creditor the whole ſum at once, at a time when being put out to intereſt it might have amounted at the end to the ſame ſum as that ariſing from the ſingle ſucceſſive payments and their intereſt; the creditor would then have received exactly the ſame advantage by the one method as by the other; and conſequently the ſubject is reduced, both in ſimple and compound intereſt, *to find in what time the whole ſum of the ſingle payments would produce the ſame amount as that which ariſes from the aggregate of each payment, together with the intereſt of each from its time of becoming due to the time of the laſt payment.*

This principle I ſhall now apply both to ſimple and compound intereſt; in order to which, let M, N, P, Q, R, &c. be the payments in ſucceſſion; t, t', t'', t''', &c. the intervals of time between the firſt and laſt, ſecond and laſt, third and laſt payments, &c. and $r =$ the rate of intereſt, alſo let x be the interval between the required or equated time, and that of the laſt payment: then becauſe $s\,(tr+1)$ is the general expreſſion for the amount of the ſum s in the time t, the ſum of the amounts aforeſaid will be $= M\,(tr+1) + N\,(t'r+1) + P\,(t''r+1) + Q\,(t'''r+1) +$, &c. which by the aforeſaid principle muſt be $= (M+N+P+$, &c.$) \times (xr+1)$ which equation being multiplied and $M+N+P+Q+$, &c. taken from both ſides, there remains $Mrt + Nrt' + Prt'' + Qrt''' +$, &c. $= (M+N+P+Q+$, &c.$)\,xr$, and dividing the whole by r we have $x = \frac{M\,t + N\,t' + P\,t'' + Q\,t''' +,\ \&c.}{M + N + P + Q +,\ \&c.}$ which gives exactly the old rule, viz. “ *Multiply each payment by its time of continuance, and divide the ſum of the products by the whole debt.*”

The ſame principle may be applied to any number of payments at compound intereſt, for $s\,r^{t}$ expreſſes the amount of any ſum s, in the time t, wherefore $r^{t}\,M + r^{t'}\,N + r^{t''}\,P + r^{t'''}\,Q +,\ \&c. + R = (M + N + P + Q +,\ \&c. + R)\,r^{x}$, conſequently $r^{x} = \frac{r^{t}\,M + r^{t'}\,N + r^{t''}\,P + r^{t'''}\,Q +,\ \&c. + R}{M + N + P + Q + \ldots\ldots + R} = a$; and hence we find $x = \log. a \div \log. r = \log. \frac{a}{r}$, which is nothing more than finding the amount of all the payments from the times they become due to the time of the laſt; then with this *amount*, and the ſum of all the payments as a *principal*, finding the *time* of continuance, according to the common rules of intereſt; and this method, with reſpect to compound intereſt, agrees exactly with Kerſey's rule.

But as “ *Mr. Profeſſor Hutton, F. R. S.*” has thought proper to condemn Kerſey's rule as *falſe*, and to give the preference to a rule of *Mr. Malcolm*'s, which he ſays is “ *the only true one*,” it will not be improper here to ſhew that Malcolm's and Kerſeys are in effect the ſame, and that both agree with the foregoing rule, when compound intereſt is allowed.

The principle on which Malcolm has founded his calculation, is the equality between the intereſt and diſcount at the equated time; but as there is apparently ſome difficulty in determining which debts are to bear intereſt, and which are to be diſcounted, he has been obliged to introduce the tedious and incorrect method of finding the time for two payments, and then making uſe of a third, and ſo on; however, this is a method which there is not the leaſt occaſion for, ſince whatever interval is aſſumed for the equated time to happen in, the inveſtigation will be exactly the ſame; and that aſſumption will have no other effect than to render the proceſs more methodical; thus if the time be ſuppoſed to fall in the interval between P and Q, and the letters to ſignify the ſame as before; then the intereſt of M for the time $t - x$; of N for the time $t' - x$, and of P for the time $t'' - x$, will be equal to the diſcount of Q for the time $x - t'''$, &c. and R for the time x, R being the laſt payment. Now $(rx - 1)\,s$ expreſſes the intereſt of any ſum s for the time x; alſo $s\,r^{-x}$ is the principal which would amount to s in th time x; conſequently the diſcount is $s - s\,r^{-x}$ or $(1 - r^{-x})\,s$: but as all the diſcounts are to be *ſubſtracted* from the ſum of the intereſts, in order to make the equation vaniſh, it is the ſame thing as *adding* them with a *contrary* ſign; but $(1 - r^{-x})\,s$

when its ſign is changed, does not differ from the expreſſion for the intereſt, except in the ſign of its index; wherefore, *if the intereſt be found with a contrary index, it will be equivalent to the diſcount with its ſign changed.*

Now the intereſt of M for the time $t - x$ is $= (r^{t-x} - 1)$ M, that of N for the time $t' - x$ is $= (r^{t'-x} - 1)$ N, and that of P for the time $t'' - x$ is $= (r^{t''-x} - 1)$ P; alſo the diſcount of Q with its ſign changed in the time $x - t''' = (r^{t'''-x} - 1)$ Q, &c. and the diſcount of the laſt payment is $(r^{-x} - 1)$ R: theſe terms being added together, and the whole made equal to nothing, alſo the equation multiplied by r^x and divided by the ſum of the payments, gives

$$r^x = \frac{r^t M + r^{t'} N + r^{t''} P + r^{t'''} Q + \text{\&c.} + R}{M + N + P + Q + \ldots\ldots R};$$

which equation is exactly the ſame as the foregoing, and the ſame concluſion would have followed had the equated time been ſuppoſed in any other of the intervals.

I cannot conclude this ſubject without obſerving, that having mentioned the above to Mr. Dalby, he ſhewed me a paper wherein he had not only deduced the very ſame concluſions, but alſo confirmed the principles on which they are founded by many ſubſtantial arguments. Hence it appears, that the common method of computing the equated time at ſimple intereſt is true, and that Kerſey's rule is true alſo in compound intereſt; As to *Profeſſor Hutton*'s aſſertions to the contrary, they have juſt as much validity as *Dr. Horſley*'s confirmation of *Stewart*'s theory of the *Sun*'s diſtance; and the ſame anſwer which *Mr. Landen* gave the *Doctor* is equally applicable to the *Profeſſor*.

Some MISCELLANEOUS PROBLEMS, *with their* SOLUTIONS.

By Reuben Burrow.

ARTICLE XII.

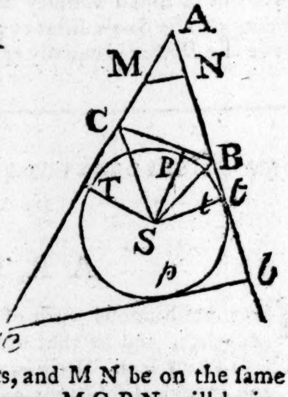
IN a poſthumous work of Dr. Simſon's (printed at Lord Stanhope's expence, and by that nobleman preſented to men of ſcience) which I have lately ſeen, there is an appendix inſerted by the editor, containing geometrical ſolutions to ſeveral problems, ſome of which are taken from Newton's Univerſal Arithmetic, and others elſewhere; but as the ſolutions there given are very long, and I had anſwers to the ſame problems by me, I flatter myſelf that to inſert them here will not be unacceptable, both on account of their ſimplicity and the impoſſibility of procuring the book aforeſaid; I was farther induced, by ſome remarks at the end of a book compiled by the *Rev. Dr Horſley, Secr. R. S.* entitled, *Apollonii Pergæi inclinationem, &c.* wherein that gentleman has been pleaſed to beſtow his cenſures very liberally on the *im-*

mane equations, and the *odious ambages* and modes of ſolution, which he ſays the modern *plebians* have *ſweat* about; and after having condeſcended himſelf to give a ſolution of Newton's 7th problem as a ſpecimen, and to refer to two propoſitions of Euclid, by which he ſays the reſt might be effected, modeſtly concludes that thoſe geometers aforeſaid, know nothing of Euclid's Data.

Whether we ought to include *Caſtillioneus* among thoſe geometers that are ignorant of the data, the doctor has not informed us; however this is certain, that he had actually ſolved Newton's Problems by thoſe very propoſitions referred to, ten years before the doctor pointed out *the ſame method*; and ſince the doctor in his propoſals for printing a new edition of Newton's works, has, in a very particular manner, informed us of his intention to give geometrical ſolutions to all thoſe problems, I had an additional motive in the clumſineſs of his method, to inſert what follows; to which if ſome (not *immane*) ſolutions be added, which are given in the London Magazine for 1775, by *Mr. George Sanderſon*, taylor, in *Doctors-Commons*, particularly a geometrical one to the 7th problem aforeſaid, which this induſtrious compiler did not ſolve without algebra, there will not remain in the *Arithmetica Univerſalis* a ſingle queſtion, relating to triangles, of any difficulty; this I point out in order to ſave the Rev. Doctor ſome trouble in his new edition; and though it has been his method hitherto, in all his *Notes, Remarks, and Compilations,* to be very ſparing of the names of thoſe authors whoſe works he has made free with, yet I hope, at the ſame time, that he will not forget to do Mr. Sanderſon the juſtice to which his merit ſo deſervedly entitles him.

PROPOSITION I. THEOREM.

If AB, AC be two lines drawn from a given point to touch a circle in T and t, and CB be any line touching the circle and intercepted between AC and AB, then will $AC + CB + BA$ be conſtant when CB is on that part of the circle next A; and $Ac + Ab - bc$ will be conſtant when cb is drawn on the contrary part For $CP = CT$, $BP = Bt$, $cp = cT$. $bp = bt$, and therefore $AT = AC + CP$, and $At = AT = AB + BP = Ac - cp = Ac - cT = Ab - bp$, conſequently $2AT = AC + CB + BA = cA + Ab - bc$.



Corollary 1. If M, N be two given points, and M N be on the ſame ſide of C B the perimeter of all the trapeziums M C B N will be invariable, or the difference between the three ſides and a fourth, &c.

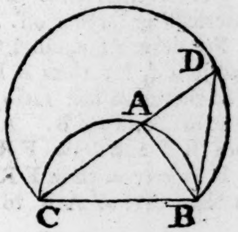
Corollary 2. Hence the ſum of the two oppoſite ſides of any quadrilateral figure, circumſcribing a circle is equal to the ſum of the other two ſides. Moreover, if there be any number of circles whatever, touching A C in the point T the perimeters of all triangles, &c. deſcribed in the ſame manner on each circle as that above, will all be equal.

Corollary 3. Hence if the perimeter, two angles and the included ſide of a trapezium be given, together with one of the oppoſite or adjacent ſides or angles, or the area, &c. the figure may be conſtructed by this and the following propoſitions.

Corollary 4. Hence Newton's 4th problem, which is the firſt in Dr. Simſon's appendix, may be generally ſolved by making $CAB =$ the given vertical angle, $AT = At =$ half the given perimeter, and drawing the circle to touch AT and At in T and t; then having deſcribed a circle from the center A, with a diſtance equal to the given perpendicular, draw a line CB to touch both circles, cutting the lines containing the given angle in C and B, then CAB will be the triangle, and the truth of the propoſition is ſelf-evident.

PROPOSITION II. PROBLEM.

If TPt be a circle given in magnitude and poſition and AT, At tangents drawn to it from a given point; it is required to draw a line CB to touch the circle ſo that the part CB intercepted between AT and At may be of a given length. See *fig.* 1.



Analyſis. Becauſe $AC + CB + BA$ is given $= 2\,AT$ and CB alſo given, $AC + AB$ is conſequently given, and the angle A; hence this conſtruction. Deſcribe on the given line CB a ſegment of a circle containing an angle equal to that made by the lines AT, At, and another on the ſame line containing half that angle, in which let CD be inſcribed equal to the given ſum of AC and AB, cutting CAB in A; then if CA, AB be taken in the firſt figure equal to the ſame lines in that annexed, the poſition of the tangent will be determined.

Corollary 1. Hence the third and eighth problems of Newton's Univ. Arith. may be generally ſolved; for the vertical angle perimeter and area being given, TA, At (ſee fig. 1.) and the angle A, are given, alſo $STAtS$ is given, and becauſe CAB is given, by ſuppoſition, $TCBtS$ is alſo given; but this laſt quantity is equal to $CB \times SP$, and SP being known CB is alſo given, and conſequently the triangle may be conſtructed by this problem.

Corollary 2. The above problem alſo includes the ſolution of Newton's tenth.

PROPOSITION III. PROBLEM.

The ſame things being given as in the laſt, it is required to draw the tangent CB ſo that its parts CP, BP may obtain a given ratio.

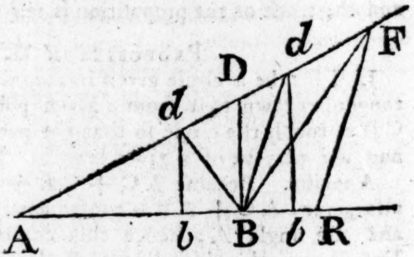
Analyſis. Becauſe ATS, AtS (ſee fig. 1.) are right angles, therefore A and TSt taken together are equal to two right angles; alſo $TSC = CSP$ and $PSB = BSt$ therefore $CSB =$ half TSt $=$ half the ſupplement of the angle A, whence the following conſtruction.

Take any given line CB and divide it according to the given ratio in P, and draw PS perpen. to CB, then on CB deſcribe a ſegment of a circle containing an angle equal to half the ſupplement of A, interſecting PS in S, and make $SBt = SBC$ and $SCT = SCP$, then tB, TC when produced to meet in A, determine a triangle ACB ſimilar to that required.

Scholium. By the foregoing problems a great number of queſtions relating to the perimeters of triangles and trapeziums may be readily reſolved, and it is worth remarking, that whatever queſtions are ſolveable thereby with reſpect to perimeters, when the tangent is drawn next the vertical point; ſimilar ones may be found the ſame way when the tangent is drawn on the part fartheſt diſtant; and the difference between the ſum of the ſides and baſe will then be concerned in like manner as the perimeter was in thoſe foregoing.

Proposition IV. Theorem.

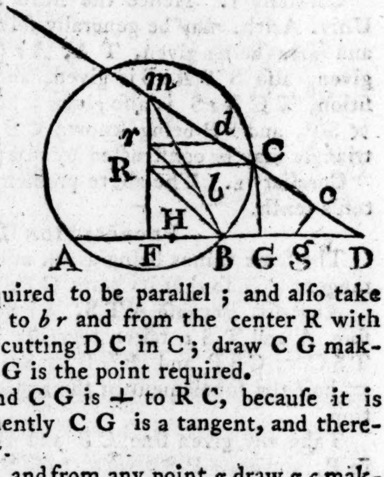
If AB, AD be two lines given in poſition meeting at A, and BD be drawn ⊥ to AB cutting AD in D, then will the ratio of AD to DB be the greateſt poſſible, and of all lines Ad and dB the ratio of thoſe which interſect neareſt D is greater than that of thoſe interſecting farther off.



For draw db parallel to DB and join d, B; then $AD : DB :: Ad : db$, but dB is greater than db, therefore the ratio of Ad to db or AD to DB is greater than that of Ad to dB. Again draw BF farther diſtant from D than d, join BF and draw $FR \parallel$ to Bd; then $Ad : dB :: AF : FR$; but FB is greater than FR, conſequently the ratio of Ad to dB is greater than that of AF to FB.

Proposition V. Problem:

A and B are two given points, and DC a line given in poſition; it is required to find a point G in AB, ſo that GC being drawn to cut DC in a given angle, the rectangle of AG and GB may be equal to the ſquare of GC.



Caſe 1. Biſect AB by the normal Fm cutting DC, and join Bm, take any point r in Fm and draw rd meeting Dm, ſo that it may be ⊥ to a line to which GC is required to be parallel; and alſo take $rb = rd$, then draw BR parallel to br and from the center R with the diſtance RB deſcribe a circle cutting DC in C; draw CG making $DCG =$ the given angle and G is the point required.

For $rd = rb \therefore RC = RB$ and CG is ⊥ to RC, becauſe it is ⊥ to rd by conſtruction, conſequently CG is a tangent, and therefore the rectangle $AGB = GC^2$.

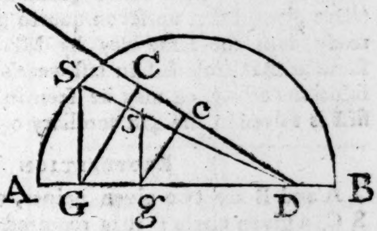
Caſe 2. Deſcribe a circle on AB, and from any point g draw gc making the given angle with DC and gs ⊥ to AB and $= gc$, continue

D,

D *s* to cut the circle in S and the normal S G will divide A B in G the point required. For $g c = g s \because G C = G S$, whence $A G B = G S^2 = G C^2$.

Corollary 1. If G C be required to be ⊥ to D C the center of the circle will fall in *m*.

Corollary 2. If from the center G with the diſtance G C a circle be deſcribed, cutting A B in H, then will all lines drawn from A and B to its circumference have the ſame ratio which A H has to H B, as is evident from the Lemma, page 337, Simpſon's Algebra.

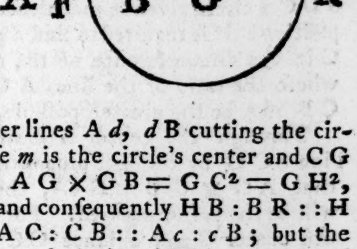


Corollary 3. If A D be a given line and B a point given, another point G may be found where $A G \times G B$ may have a given ratio to $G D^2$, by taking any line *g*D, deſcribing a ſemicircle thereon, and in it taking $g c^2$ to $g D^2$ in the given ratio, then drawing F *m* ⊥ to and biſecting A B, and cutting D C in *m*, then taking $m C = m B$ and drawing C G parallel to *c g*, and G will be found; for $G C^2 : G D^2 :: g c^2 : g D^2$ and *m* being the center C G is a tangent to the circle, and conſequently its ſquare $= A G \times G B$; wherefore $A G B : G D^2 :: g c^2 : g D^2$, viz. in the given ratio. Alſo in the ſecond figure $A G B = G S^2 = G C^2$ and $G C^2$ is to $G D^2$ in the given ratio, therefore $A G \times G B$ is to $G D^2$ in the ſame ratio; and in a ſimilar manner may be the reſt of Apollonius's problems on Determinate Section be reſolved, as will be evident to any perſon that takes the trouble of obſerving the method which Mr. *Wales* took in collecting his book thereon from the ſolutions that had been given before by Mr. *Simpſon* and *Snellius*.

PROPOSITION VI. PROBLEM.

C D is a line given in poſition and A, B two given points: it is required to find a point C in the line C D, where the ratio of A C to C B may be the greateſt poſſible.

Biſect A B by the perpendicular F *m*, meeting D C in *m*, and take $m C = m B$, then C will be the point required.



For draw C G ⊥ to D C meeting A B in G, and with the center G and diſtance G C deſcribe a circle, which of courſe touches D C in C, alſo join A C, C B and drew any other lines A *d*, *d* B cutting the circle in *c*, and D E in *d*; then becauſe *m* is the circle's center and C G ⊥ to *m* C, C G is a tangent, and $A G \times G B = G C^2 = G H^2$, therefore $A G : G H :: G H : G B$, and conſequently $H B : B R :: H A : A R$; wherefore $A H : H B :: A C : C B :: A c : c B$; but the ratio of A *c* to *c* B is greater than that of A *d* to *d* B by prop. 4, and therefore the ratio of A C to C B is alſo greater than that of Ad to dB, conſequently is the greateſt poſſible.

Corollary 1. The other interſection of the circle gives another point, but the method is the ſame for all caſes.

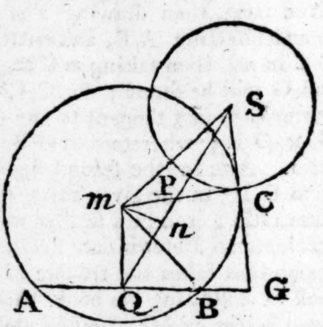
Corollary 2. Hence, if there be an indefinite number of right lines parallel to C D, the locus of all the points C will be an hyperbola; for F m is given in poſition, and the diſtances m C are ſet off in a direction making a conſtant angle with F m and equal to m B.

Scholium. The above problem is Dr. Simſon's 5th, the ſolution there given takes up ſeven quarto pages: as to the 4th it has been already done the ſame way by Mr. Simpſon; the 2d. and 3d. are the ſame as that propoſed in laſt year's Diary by Mr. Sanderſon, different ſolutions of which may be ſeen in the anſwers for this year; and the firſt is ſolved in the 4th corollary of the firſt propoſition.

PROPOSITION VII. LEMMA.

A and B are two given points, and S C a given circle: it is required to find the point G in A B, ſo that G S being drawn to the center and meeting the circumference in C, the ſquare of C G may be equal to the rectangle A G and B G.

Draw Q m ⊥ to and biſecting A B join S Q which biſect in P, and take P n ſo that $2 P n \times S Q = Q B^2 + S C^2$, then nm drawn ⊥ to Q S gives m the center of a circle, which being deſcribed with the diſtance m B cuts the circle S C in C, then C S being drawn, cuts A B in G the point required.

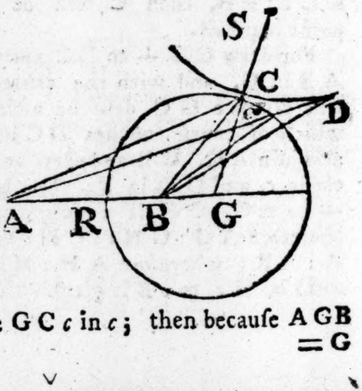


For $2 P n \times S Q = S n^2 - n Q^2 = S m^2 - m Q^2 = Q B^2 + S C^2$, therefore $S m^2 - S C^2 = Q B^2 + Q m^2 = m B^2$; but m B is by conſtruction $= m$ C, therefore $S m^2 - S C^2 = m C^2$, conſequently m C S is a right angle, and C G a tangent to the circle A B C, whence it follows that $A G \times G B = G C^2$.

PROPOSITION VIII. PROBLEM.

A and B are two given points, and S D C a circle given in magnitude and poſition: it is required to find a point C in the circumference of the circle where the ratio of the lines A C and C B may be the greateſt poſſible.

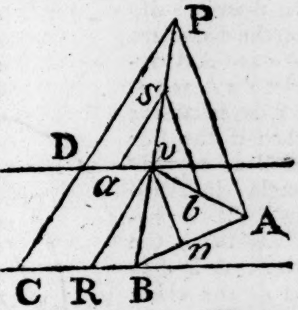
Through the center S draw the line S C G by the laſt propoſition ſo that $A G \times G B = G C^2$, then C will be the point required. For with the center G and diſtance G C deſcribe a circle, and draw any other lines A D, D B, the firſt cutting the circle G C c in c; then becauſe A G B



= G

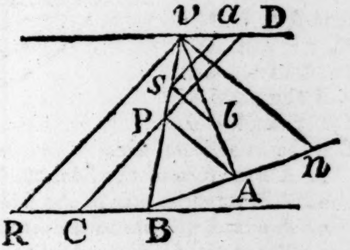
$= GC^2 = GR^2$, $AC : CB :: Ac : cB$; but the ratio of Ac to cB is greater than that of AD to DB, conſequently the ratio of AC to CB is greater than that of AD to DB, and therefore the ratio of AC to CB is the greateſt poſſible.

Proposition VIII. Problem.



B v and B C are two lines given in poſition and A a given point: it is required to find the point P in the line B v ſo that A P being joined, and P C drawn parallel to a line given in poſition, the ratio, ſum, or difference of A P and P C may be given.

1. Draw R v parallel to the line given in poſition, and at ſuch a diſtance that v R may be equal to the given ſum or difference, join A v and A B, and take any point a from which draw as parallel to R v, and take $sb = sa$ cutting A v in b, and draw A P parallel to bs, meeting B v in P, then will the ſum or difference of A P and P C be equal to v R. For $vs : vP :: as : PD :: sv : AP$, therefore $AP = PD$ and $CP \mp PA = CD = Rv$.



2. For the ratio; take vn to v R in the given ratio which AP is required to have to P C, and parallel to vn draw A P, and the thing is done; for $Bv : BP :: vn : PA :: vR : PC$, therefore $vn : vR :: PA : PC$.

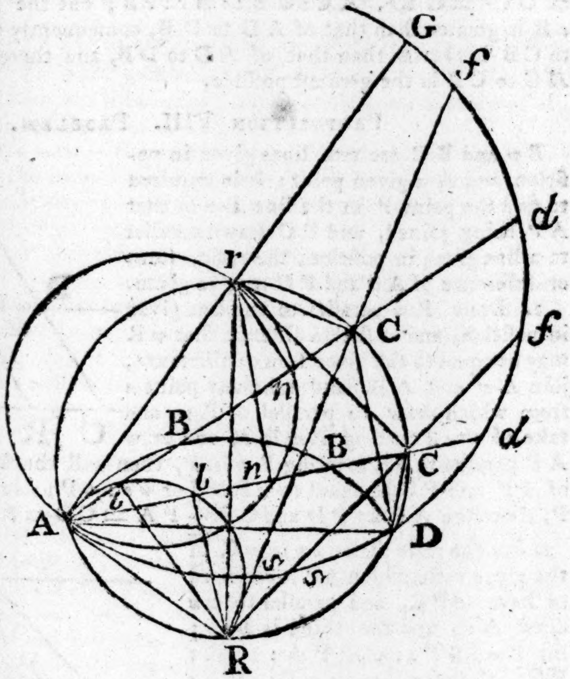
Corollary 1. Hence the 48th problem of Newton's Algebra may be ſolved geometrically; by continuing A B (ſee Newton's figure) and finding the point E in B C ſo that F E being drawn perpen. to the horizon meeting A B produced in P, the ratio of A E to E P may be as the weight D to the weight E: for if P E repreſent the weight E, B E will repreſent its force down the plane, B C, and as A E repreſents the force of D, E B repreſents its force in the direction E B, and conſequently the weights are in equilibrio.

Corollary 2. If the ratio of C P to P A be required to be the greateſt poſſible, let A P be perpen. to A B: the reaſon is evident from the 4th propoſition.

Scholium. The application of this problem is very extenſive, particularly in mechanics, wherein lines are often required to be drawn parallel to the direction of gravity, &c. the problems of gunnery (abſtracting the air's reſiſtance) may alſo be conſtructed by it, in a much ſimpler manner than any publiſhed hitherto, as I ſhall ſhew hereafter.

Proposition IX. Theorem.

If A C D R be a circle, A D a chord, R r a diameter ⊥ to A D, and A C D, A C D triangles in the ſegment A C D; then if from the



the centers r and R with the diſtances rA and RA circles $Asdf G$ and ABD be drawn; alſo on the diameters Ar and AR circles rnA and AbR be deſcribed; then if the ſide AC of any triangle inſcribed in ACB be produced to cut the circle $ADdG$ in d, the circle Anr in n, the circle ABD in B and the circle AbR in b; alſo if Rb, DB, rB, rn and RB be drawn, then will the parts of the triangles ACD be as follows, viz.

1. $Ad =$ ſum of the ſides $= AC + CD$
2. $AB =$ difference of the ſides
3. $An =$ half ſum of the ſides $= Cb$
4. $Ab =$ half difference of the ſides $= nC$
5. $Cs^2 =$ rectangle of the ſides.
6. $bRA =$ half difference of the angles at the baſe $= rAC$

1. For the angle $ArD = 2\,AGD$ becauſe r is the center, therefore $ACD = 2\,AdD = AdD + CDd$, conſequently $Cd = CD$ and $AC + CD = Ad$.

2. In order to avoid the multiplicity of lines, ſuppoſe AB drawn through the point biſecting the arch ABD, and DR joined, then the proof that $CD = CB$, and conſequently that $AB = AC - CB$ will be thus: $AB = BD$ and $RB = RD$, therefore $ARB = BRD$ and $RBD = RDB$; but $CBD + ABD = 2$ right angles $= CBD + 2\,RDB$, therefore $CBD + 2\,RDB = CDB + 2\,RDB$, and conſequently $CB = CD$ in this caſe; but the angles ABD and ACB are conſtant, and conſequently Cm is $= CD$ in every caſe.

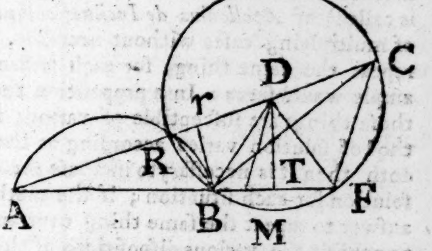
3. Becauſe Anr is a right angle and r the center of $ADdG$ ∵ $An = nd =$ half $AC + CB$, alſo $Ab = bB$ and $BC = Cd$ ∵ $bA =$ half $Ad = An = nd$.

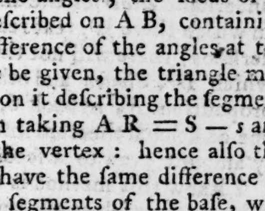
4. $Ab =$ half $AB =$ half $AC - CD$.

5. $AC \times Cd = sC \times Cf = Cs^2$, becauſe $sC = Cf$ by Euclid. prop. 3.

6. A R *b* = A D B = C D A − C D B = C D B + B D A (C B D) B D A − C A D = C D B − C A D ∵ 2 A D B = C D A − C A D, conſequently A R *b* equal half the difference of the angles at the baſe equal *r* A C, &c.

In the ſame manner if A D F be any triangle, and from the center D a circle be deſcribed with the diſtance D F; the lines being joined as in the figure, and D M parallel to C F; then it is evident that A B is the difference of the ſegments of the baſe, and A R that of the ſides, and becauſe A C F = half A D F, D M biſects A D F, and therefore M D T = half the difference of the angles at the baſe; but if B *r* be perpen. to A D, A B *r* = A D T and A B R = A C F = A D M, therefore R B *r* = M D T, conſequently A R B = R B *r* + R *r* B = 90° + half the diff. of the angles at the baſe, and therefore when the difference of the ſegments of the baſe is given, and the difference of the angles; the locus of all the points R will be a circular ſegment deſcribed on A B, containing an angle equal to 90 degrees + half the difference of the angles at the baſe; hence if S — *s*, A — *a*, and *m* — *n* be given, the triangle may be conſtructed, by taking A B = *m*—*n*, and on it deſcribing the ſegment containing the angle 90 + $\frac{1}{2}$ (A — *a*) then taking A R = S — *s* and making R B D = B R D and D will be the vertex: hence alſo the locus of the vertices of all triangles which have the ſame difference of the angles at the baſe, and difference of the ſegments of the baſe, will be a circle; for as A R B is conſtant, and B R *r* alſo, and R B C a right angle, B C R is alſo conſtant as well as its double B D R, and R B D; wherefore B R is in a conſtant ratio to B D and B C, and conſequently the points D and C are in circles; and hence a great number of caſes of triangles may be conſtructed; for inſtance, if *m* — *n*, A — *a*, and either S, P, or S : *s* or S × *s* be given, and many others; and ſimilar methods may be uſed when A R is conſtant and A B and the other parts variable. Hence alſo if M G = M F, A G : A R :: A R : A B for A F : A D + D F :: A R : A B :: A M : A D :: F M : F D, therefore A R : A B :: (A M — F M) A G : (A D — T D) A R. The following problem will alſo be uſeful in ſeveral conſtructions, viz. if A B C, A D C be two given circles interſecting in A and C, and it be required to draw A B D ſo that the rectangle of A B and B D may be given: becauſe the angles B and D are conſtant; the ratio of B C to B D is given, and conſequently the ratio of A B × B C to A B × B D, but A B × B C = B E × by the diameter of the circle, and this being given, B E is alſo given; in the ſame manner may A B be drawn to have a given ratio to B D.





Some parts of this problem are not new, but were here brought together into one view for the ſake of making references in order to ſhorten the ſolutions of problems.

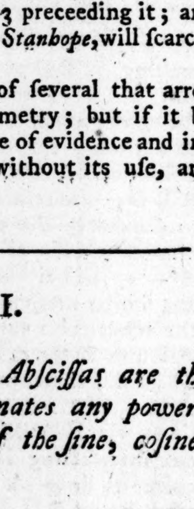
Scholium. In the foregoing propoſitions I have ſeldom given ſolutions to more than one caſe; there are ſome that admit of more caſes, but the method laid down will be applicable to the reſt with ſo little alteration, that I did not think it neceſſary to be more particular. I do not doubt but ſuch a proceedure will be looked upon as deviating from geometrical ſtrictneſs by ſuch as have formed their ideas of the method of the ancients from the ſpecimen given by the *Reſtorer* (as he is called) of *Apollonius de Inclinationibus*; however I cannot ſee the uſe of multiplying caſes without neceſſity, nor what end it can anſwer to repeat the ſame thing, for each trifling alteration, when a ſingle example would ſerve: In a propoſition there are certain things given, and thoſe things are ſuſceptible of various ſituations; now either the method of ſolution varies according as thoſe ſituations vary, or not; if it doth then it is neceſſary to increaſe the number of caſes till there be a ſolution for each ſituation; if the method do *not* vary, what end can it anſwer to repeat the ſame thing over and over for no other purpoſe but to exhibit the various diſpoſitions of the data; when the ſame end may be fully accompliſhed by only increaſing the number of diagrams? Nay I do not even ſee any neceſſity for this laſt; *Euclid* does not uſe it, and if by "the inclination of two ſtreight lines which meet together," we underſtand *either* of the angles made at the point of interſection, (a ſenſe in which there is great reaſon to believe that *Euclid* intended to be underſtood) there will not then be the leaſt occaſion for ſeveral additions which *Dr. Simſon* and others have made to the elements; for inſtance, propoſition A, in book 6, will be included in prop. 3 preceeding it; and the additional theorem inſerted in the data by *Lord Stanhope*, will ſcarcely amount even to a ſecond caſe of prop. 97. &c.

I know this method is contrary to the practice of ſeveral that arrogate to themſelves all knowledge in ancient geometry; but if it be agreeable to common ſenſe, and give the ſame degree of evidence and inſtruction in a leſs compaſs, it certainly cannot be without its uſe, and may, for that reaſon, at leaſt be tolerated.

ARTICLE XIII.

Of finding the Areas of Curves whoſe Abſciſſas are the ſame as thoſe in a Circle, and their Ordinates any powers of the correſponding Arc or Multiples of the ſine, coſine, &c. By Mr. William Wilkin.

1. LET AEB be a ſemicircle whoſe diameter is AB and center C, and from B, C, and any point M erect the perpendiculars BD, CF, and MN; and let MN be equal to any power of the arc A*e*; to find the quadrature of the ſpace ANM or the fluent of $z^m \dot{x}$, (putting AB $= 2a$ M*e* $= v$, AM, $= x$, MN $= y$, A*e* $= z$, and the index of the power $= m$.)



Aſſume the fluent $= z^m x + q$, then will $z^m \dot{x} + m z^{m-1} \dot{z} x \times \dot{q} = z^m \dot{x}$, and therefore $\dot{q} = - m z^{m-1} \dot{z} . x = - \frac{a m z^{m-1} x \dot{x}}{\sqrt{2 a x - x^2}} = - a m z^{m-1} \times (\dot{z} - \dot{v})$ and by taking the fluents $q = - a z^m +$ fluent $a m z^{m-1} \dot{v}$. Again aſſume the fluent of $a m z^{m-1} \dot{v} = a m z^{m-1} v + r$, then will $a m z^{m-1} \dot{v} + a m . \overline{m - 1} . z^{m-2} \dot{z} . v + \dot{r} = a m z^{m-1} \dot{v}$, therefore $\dot{r} = - a m . \overline{m - 1} . z^{m-2} \dot{z} v = - a^2 m . (m-1) . z^{m-2} \dot{x}$, then again aſſume the fluent $r = - a^2 m . \overline{m - 1} . z^{m-2} x + s$, and by proceeding as before $\dot{s}$ will be $= a^2 m (m-1) . (m-2) . z^{m-3} \dot{z} . x = \frac{a^3 m . (m-1) . (m-2) . z^{m-3} x \dot{x}}{\sqrt{2 a x - x^2}} = a^3 m . (m-1) . (m-2) \times z^{m-3} \times (\dot{z} - \dot{v})$ and therefore $s = a^3 m . (m-1) . z^{m-2} -$ fluent $a^3 m . (m-1) . (m-2) . z^{m-3} \dot{v}$. Whence again aſſume $- a^3 m . (m-1) . (m-2) . z^{m-3} v + t$ for the fluent ſo will $\dot{t} = a^3 m . (m-1) . (m-2) . m - 3 . z^{m-4} \dot{z} . v = a^4 m . (m-1) . (m-2) . (m-3) . z^{m-4} \dot{x}$ and $t = a^4 m . (m-1) . (m-2) . (m-3) . z^{m-4} x + v$: conſequently (the law of continuation being manifeſt) the fluent of the given expreſſion will be $= z^m x - a z^m + a m z^{m-1} v - a^2 m . (m-1) . z^{m-2} . x + a^3 m . (m-1) . z^{m-2} - a^3 m . (m-1) . (m-2) . z^{m-3} v + a^4 m . (m-1) \ldots\ldots (m-3) . z^{m-4} x - a^5 m \ldots . (m-3) . z^{m-4} +$, &c.

Corollary 1. If w be put equal $a - x$ (the coſine of the arc z) in the above expreſſion, it will become $= - w z^m + a m z^{m-1} v + a^2 m . (m-1) . z^{m-2} w - a^3 m . (m-1) . (m-2) . z^{m-3} v - a^4 m . (m-1) \ldots (m-3) . z^{m-4} w +$, &c. the ſame as found at page 390 of Mr. Simſon's Flux. 2d edit.

Corollary 2. If $m = 1$. then ſhill the fluent of $z \dot{x}$ or the area of the curve ſpace A N M (whoſe ordinate M N is = circular arc A e,) $= z x - a z + a v = z x - a z + a \sqrt{2 a x - x^2}$; whence $x = a$, or the ordinate paſſes through the center C, the area A F C = A C², and when $x = 2 a$. the area of the whole curve A D B A = A C × circumference A E B = the circle whoſe diameter is A B.

Corollary 3. If $m = 2$, the area A N M $= z^2 x - a z^2 + 2 a z v - 2 a^2 x = ($ when $x = 2 a) \; a z^2 - 4 a^3 = \mathrm{AC} \times (\overline{\mathrm{AEB}})^2 - \overline{\mathrm{AB}})^2$ = the exceſs of the circle whoſe diameter is A B above twice the ſquare of the rad, A C, for the whole ſpace A D B.

Corollary 4. If $m = 3$ the area $= z^3 x - a z^3 + 3 a z^2 v - 6 a^2 z x + 6 a^3 z - 6 a^3 v = ($when $x = 2 a) \; a z^3 - 6 a^3 z$.

Corollary 5. If $m = 4$ the area $= z^4 x - a z^4 + 4 a z^3 v - 12 a^2 z^2 x + 12 a^3 z^2 - 24 a^3 z v + 24 a^4 x = ($when $x = 2 a) \; a z^4 - 12 a^3 z^2 + 48 a^3$, &c. &c.

2. Suppoſe the ordinate of the curve M N to be always equal to a rectangle of any power of the arc and the verſed ſine $= z^m x^n$ to find the curve of the ſpace A M N or the fluent of $z^m x^n \dot{x}$. Aſſume the fluent $= \frac{z^m x^{n+1}}{n+1} + r$, then in fluxions $\dot{r} = -\frac{m x^{n+1} z^{m-1} \dot{z}}{n+1}$ $= -\frac{m x^{n+1} z^{m-1}}{n+1} \times \frac{a \dot{x}}{\sqrt{2ax - x^2}} = -\frac{a m z^{m-1}}{n+1} \times \frac{x^{n+1} \dot{x}}{\sqrt{2ax - x^2}}$ $= -\frac{a m z^{m-1}}{n+1} \times \frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$. Again aſſume the fluent $r = -\frac{a m z^{m-1} A}{n+1} + s$ (putting $A =$ fluent $\frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$,) then will $\dot{s} = \frac{a m . \overline{m - 1} . z^{m-2} \dot{z} A}{n+1}$. Now by finding the value of A or the flu. of $\frac{x^{n+\frac{1}{2}} \dot{x}}{\sqrt{2a - x}}$ (which may be eaſily had from Emer. Table, Form 11. when n is any affirmative whole number, and thence s, or the fluent of $\frac{a m . m - 1 . z^{m-2} \dot{z} A}{n+1}$ and aſſuming other variable determinate values as before the required fluent of $z^m x^n \dot{x}$ will therefore be evidently $\frac{1}{n+1} \times : z^m x^{n+1} - a m z^{m-1} A + a m . (m - 1) . z^{m-2} B - am . m - 1 . m - 2 . z^{m-3} C$, &c. A being as above, $B =$ fluent of $A \dot{z}$, $C =$ fluent of $B \dot{z}$. &c.

But as this cannot be purſued in a general manner by this method it will be neceſſary to ſhew how to proceed in particular caſes, when m and n are given in numbers.

Thus. 1. If m and n each equal 1, then will $\dot{A} = \frac{x^{\frac{3}{2}} \dot{x}}{\sqrt{2a - x}}$, and by taking the fluents $A = \frac{3}{2} a z - \frac{3a + ax}{2} \times \sqrt{2ax - x^2}$ therefore the area will be $= \frac{z x^2}{2} - \frac{3az}{4} + \frac{3a^2 + ax}{4} \sqrt{ax - x^2}$ which when x is $= 2a$ becomes $= \frac{5a^2 z}{4}$ the whole area of the curve. Or the fluent of the expreſſion $z x \dot{x}$.

2. If $m = 1$ and $n = 2$ then will $\dot{A} = \frac{x^{\frac{5}{2}} \dot{x}}{\sqrt{2a - x}}$ and conſequently

A

$A = \frac{5}{2}a^2 z - \overline{\frac{1}{3}x^2 + \frac{5}{6}ax + \frac{5}{2}a^2} \times \sqrt{2ax - x^2}$ and the area $= \left(\frac{z^m x^{n+1}}{n+1} - \frac{amz^{m-1}A}{n+1}\right)\frac{zx^3}{3} - \frac{5a^3z}{6} + \overline{\frac{ax^2}{9} + \frac{5a^2x}{18}}$ $\overline{+\frac{5a^3}{6}}\sqrt{2ax - x^2}$ — (when $x = 2a$) $\frac{11}{6}a^3 z$ for the whole ſpace A D B or the fluent of $z x^2 \dot{x}$.

3. If $m = 2$ and $n = 1$, then $\dot{A} = \frac{x^{\frac{3}{2}}\dot{x}}{\sqrt{2a - x^2}}$ and $\dot{s} = \frac{3a^2z\dot{z}}{2} - \frac{3a^2 + ax}{2}\sqrt{2ax - x^2} \times (\dot{z}) \frac{a\dot{x}}{\sqrt{2ax - x^2}}$, hence by taking the fluents $s = \frac{3a^2z^2}{4} - \frac{3a^3x}{2} - \frac{a^2x^2}{4}$, therefore the area $\frac{z^m x^{n+1}}{n+1} - \frac{amz^{m-1}A}{n+1} + s) = \frac{z^2x^2}{2} - \frac{3}{4}a^2z^2 + \frac{\overline{3a^2z + axz}}{2}\sqrt{2ax - x^2} - \frac{3a^3x}{2} - \frac{a^2x^2}{4} =$ (when $x = 2a$) $\frac{5}{4}a^2z^2 - 2a^4$.

4. If m and n each equal 2, then $\dot{A} = \frac{x^{\frac{5}{2}}\dot{x}}{\sqrt{2a - x}}$ and $\dot{s}$ $= \left(\frac{am.\overline{m-1}.z^{m-2}A}{n+1} \times \dot{z}\right) = \frac{5}{3}a^3z\dot{z} - \frac{2}{9}a^2x^2\dot{x} - \frac{5}{9}a^3x\dot{x} - \frac{5}{3}a^4\dot{x}$, whence $s = \frac{5}{6}a^3z^2 - \frac{2}{27}a^2x^3 - \frac{5}{18}a^3x^2 - \frac{5}{3}a^4x$ and the area $= \frac{z^2x^3}{3} - \frac{5}{6}a^3z^2 + \overline{\frac{2}{9}azx^2 + \frac{5}{9}a^2zx + \frac{5}{3}a^3z}\sqrt{2ax - x^2} - \frac{2}{27}a^2x^3 - \frac{5}{18}a^3x^2 - \frac{5}{3}a^4x =$ (when $x = 2a$) $\frac{11}{6}a^3z^2 - \frac{136}{27}a^5$.

5. If $m = 3$ and $n = 2$, $\dot{A} = \frac{x^{\frac{5}{2}}\dot{x}}{\sqrt{2a - x}}$ as before, ($\dot{s} = 5a^3z^2\dot{z} - \frac{2}{3}a^2x^2\dot{x} - \frac{5}{3}a^3x\dot{x} - \frac{5}{2}a^4\dot{x}$ and $\dot{t} = -\frac{5}{2}a^3z^2\dot{z} - \frac{2}{9}a^3x^3\dot{x} - \frac{5}{6}a^4x^2\dot{x} - 5a^5x\dot{x}$), or $B = \frac{5}{4}a^2z^2 - \frac{1}{9}ax^3 - \frac{5}{12}a^2x^2 - \frac{5}{2}a^3x$ and $C = \frac{5}{12}a^2z^3 - \frac{1}{36}a^2x^4 - \frac{5}{36}a^3x^3 - \frac{5}{4}a^4x^2$, whence by ſubſtituting theſe values in the above general expreſſion, the area becomes $= \frac{z^3x^3}{3} - az^2 . A + 2azB - 2aC = \frac{z^3x^3}{3}$ $\frac{5}{6}a^2x^3 + \frac{1}{3}ax + \frac{5}{6}a^2x + \frac{5}{2}a^3 \times z^2y(-\frac{2}{9}ax^3 - \frac{5}{6}a^3x^2 - 5a^4x) \times z + \frac{1}{18}a^3x^4 + \frac{5}{18}a^4x^3 + \frac{5}{2}a^5x^2$.

6. If m and n each equal 3; then $\dot{A} = \frac{x^{\frac{7}{2}}\dot{x}}{\sqrt{2a - x}}$ or $A = \frac{35}{8}a^4z - \frac{1}{4}x^3y - \frac{7}{12}ax^2y - \frac{35}{24}a^2xy - \frac{35}{8}a^3y$, $B = \frac{35}{16}a^4x$

$-\frac{1}{16}ax^4 - \frac{7}{36}a^2x^3 - \frac{35}{48}a^3x^2 - \frac{35}{8}a^4x$, C $= \frac{35}{48}a^4z^3 - \frac{1}{80}a^2x^5 - \frac{7}{144}a^3x^4 - \frac{35}{144}a^4x^3 - \frac{35}{16}a^5x^2$, and D $= \frac{35}{172}a^4x^4 - \frac{1}{483}a^3x^6 - \frac{7}{720}a^4x^5 - \frac{35}{456}a^5x^4 - \frac{35}{48}a^6x^3$ ſubſtitute theſe values in the theorem and it will give $\frac{z^3x^4}{4} - \frac{35}{32}a^5\dot{z}^3 + \frac{3}{16}ax^3 + \frac{7}{16}a^2x^2 + \frac{35}{32}a^3x + \frac{105}{32}a^4 \times z^2y - \frac{3}{32}a^2x^4 - \frac{7}{24}a^3x^3(-\frac{55}{32}a^4x^2 - \frac{105}{16}a^5x) \times z + \frac{3}{160}a^3x^5 + \frac{7}{96}a^4x^4 + \frac{35}{96}a^5x^3 + \frac{105}{32}a^6z^2$ for the area, &c. &c.

Suppoſe the ordinate of the curve M N to be always equal to the rectangle of any power of the arc, verſed ſine and ſine $= z^m x^n v^r$ to find the area of the ſpace A M N or the fluent of $z^m x^n v^r \dot{x}$.

Aſſume $\frac{z^m v^r x^{n+1}}{n+1} + s$ for the fluent, then $\dot{s} = -\frac{1}{n+1} \times : m x^{n+1} v^r z^{m-1} \dot{z} + r x^{n+1} z^m v^{r-1} \dot{v} = -\frac{1}{n+1} \times : m z^{m-1} x^{n+1} v^r \times \frac{a\dot{x}}{v} + r z^m x^{n+1} v^{r-1} \dot{v} = -\frac{1}{n+1} \times : r z^m x^{n+1} v^{r-1} \dot{v} + a m z^{m-1} . x^{n+1} . v^{r-1} \dot{x}$. Again aſſume $s = -\frac{1}{n+1} \times a m z^{m-1} A + r z^m B + t$ (putting A = the fluent of $x^{n+1} v^{r-1} \dot{v}$ and B = fluent of $\frac{\dot{x}}{v}$ A) then will $\dot{t} = \frac{1}{n+1} \times : am . \overline{m-1}\, z^{m-2} A \dot{z} + r m z^{m-1} B \dot{z}$. Aſſume now this fluent $= \frac{1}{n+1} \times : am . \overline{m-1}\, z^{m-2} C + r m z^{m-1} D + u$ (putting C = fluent of $\dot{z}$ A and D = that of $\dot{z}$ B) by proceeding as before, and aſſuming another value for u, the law of continuation will be evident, putting again in this value E = flux. $\dot{z}$ C and E = that of $\dot{z}$ D; The fluent will therefore be generally expreſſed thus $\frac{z^m v^r x^{n+1}}{n+1} - \frac{1}{1+n}$ $(r z^m A + a m z^{m-1} B + r m x^{m-1} C + am . \overline{m-1} . z^{m-2} D) - (rm . \overline{m-1} . z^{m-2} E + am . \overline{m-1} . \overline{m-2} . z^{m-3} F +$ &c.)

To exemplify this theorem, take the 1 Exam. given by Mr. Simpſon to his ſolution of the ſame problem at page 393 of his fluxions. Then will $m = 1 . n = 0$, and $r = 1$; whence $zvx - zA +$ fluent $\dot{z}$ A $= zvx - \frac{1}{2}zx(xv - az + av) - \frac{1}{2}ax^2 +$ fluent of $\frac{1}{2} : (vz - az + av) \times \frac{av}{x} = zvx - \frac{1}{2} \times ($[illegible]$vx + a\dot{z}^2 - avz - ax^2)$

$+\frac{1}{4}\times(ax^2-az^2)+\frac{1}{2}a^2x=\frac{1}{2}zvx+\frac{1}{4}az^2-\frac{1}{2}avx-\frac{1}{4}ax^2+\frac{1}{2}a^2x=\frac{1}{4}az^2-\frac{1}{2}avz+\frac{1}{3}zvz+\frac{1}{4}av^2$.

N. B. In the above quoted ſolution x is the coſine, and in this x is the verſed ſine, if therefore in the concluſion $a-x$ be ſubſtituted for the verſed ſine it will appear to correſpond with the above.

Scholium. From theſe may be had the ſolutions of ſeveral queſtions that have been propoſed in the annual publications relating to circular areas and cycloidal ſpaces. The method may be purſued much farther, and extended to different enquiries of a ſimilar nature, where arcs of any kind, hyps. logs. &c. are involved with the fluxions of their contemporaneous part, though perhaps not in ſo conciſe a manner as by the method of aſſuming a ſeries with unknown coefficients, &c. yet to beginners it will appear much more plain and intelligible, for whoſe uſe and improvement the application of the theorems is intended.

New MATHEMATICAL QUESTIONS *to be anſwered in next Year's* DIARY.

[1] XIV. QUESTION, *by Mr.* Edward Boucher.

GIVEN $\left.\begin{array}{l}x^3y+y^3x=a\\x^6y^2+y^6x^2=b\end{array}\right\}$ to find x and y.

[2] XV. QUESTION, *by Mr.* Fininley.

GIVEN the difference of the ſegments of the baſe, the difference of the angles at the baſe, and the rectangle made by one of the ſides, and a line to which the other ſide hath a given ratio: to find the triangle.

[3] XVI. QUESTION, *by Mr.* John Lynn, *of* Sunderland

LET the given line A B be perpendicular to the the indefinite line A Q, and drawing any right line B E from the fixt point B, to cut A Q in E, and taking E C thereon in a given ratio to E A; it is required to find the nature of the curve, &c.

[4] XVII. QUESTION, *by Mr.* George Sanderſon.

IN a triangle A C B, the baſe A B is given, and the difference of the ſides A C and C B: it is required to conſtruct the triangle geometrically when the difference of A D and D C is the leaſt poſſible; C D being drawn from the vertex of the triangle to meet the baſe in a given angle.

[5] XVIII. QUESTION, *by Mr,* Iſaac Dalby.

IF A B and A C be tangents to a given circle meeting in the given point A, and from this point with a given diſtance a circle be deſcribed; it is required to draw a tangent to the firſt circle, cutting the laſt in Q, and the tangents in N and R, ſo that N Q may be equal to Q R,

[6] XIX. QUESTION, *by Mr.* William Wilkin.

IF a caſk is formed by the revolution of the quadratrix of Dinoſtratus, about the diameter of the generating ſemicircle; it is required to determine the number of ale gallons it will contain when the bung and head diameter are 40 and 30 inches reſpectively.

[7] XX. QUESTION, *by Mr.* D. Cunningham.

REQUIRED the ſum of any number of terms of the infinite ſeries $\frac{2\cdot4\cdot6}{3}+\frac{4\cdot6\cdot8}{3\cdot3}+\frac{6\cdot8\cdot10}{3\cdot3\cdot3}+$, &c.

[8] XXI. Question, *by Mr.* Thomas Moſs.

IF from the extremities S and V of the baſe of a triangle STV two lines be drawn through a given point N meeting TV and ST in C and A; and the line TN be joined meeting AC in B; alſo if from A and C parallel lines be drawn meeting the baſe in M and P; then will AB be to BC as AM to CP: required the demonſtration?

[9] XXII. Question, *by Mr.* Thomas Todd.

A Has 1000 l. due from B one year hence, beſides D pounds ſeven years hence; to determine D pounds, with the equated time, as given by Malcolm's method, ſo that A may gain 20 l. more by this equatement than if he had received his money as it came due, 5 per cent. per annum ſimple intereſt being allowed to both A and B.

[10] XXIII. Question, *by Mr.* Jeremiah Ainſworth.

HAVING a circle given in magnitude and poſition, the center of which is ſituated in a line biſecting an angle made by two lines given in poſition: it is required to draw a tangent to the circle, ſo that the ſegment intercepted between theſe two lines may be of a given length.

[11] XXIV. Question, *by the Rev. Mr.* Crakelt.

GIVEN the triangle ABC, and the poſition of the point P in the ſide BC; it is required to draw the line DE through the ſaid point, meeting AC in D, and AB produced in E, in ſuch ſort, that the ſum of tho areas of the two triangles PCD and PBE may be equal to the area of the trapezium ADPB.

N. B. This is Queſtion 337 of the Gent. Diary.

[12] XXV. Prize-Question, *by* Reuben Burrow.

IF AB, AC and AN be lines given in poſition, meeting in the point A, and P a given point; it is required to draw the line PE, meeting AB in D and AC in E, in ſuch a manner, that EF being drawn parallel to NA to meet AB in F, the perimeter of the triangle DEF may be the greateſt poſſible; without algebra.

Whoever gives the beſt Solution to this Queſtion, before Candlemas-day, *ſhall receive Six* Diaries *and a Silver Medal; and the Perſon who gives the next beſt Solution ſhall be intitled to the Prize of Twelve* Diaries.

⁂ The Author of the Ladies and Gentlemens Diary does not preſume to reſtrict any of his Correſpodnents from contributing in any manner they may think proper to "*other publications of a ſimilar kind,*" as he will always ſtudy to "*give the preference*" to merit only.

Juſt publiſhed, Price 2s. 6d.

Dedicated, by Permiſſion, to the Hon. Commiſſioners of Exciſe.

THE Deſcription and Uſe of a new invented Inſtrument for taking the true internal Diameter between the Bung and Head of any lying Caſk; together with a new and extenſive Table, ſhewing the internal length; by Means of which and the ſaid middle Diameter, the Content of any Caſk of whatever Curvature may be truly and readily obtained.

By Mr. THOMAS MOSS.

Correſpondents are deſired to ſend their Letters directed to be left at Mr. Carnan's, *No.* 65, *St. Paul's Church-Yard; or to Mr.* Burrow, *Mathematical Maſter of the Drawing Room at the Tower*